Elimination Method for Parallelization of Flexible Multibody System Dynamics with Kinematical Loops

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ABSTRACT — *The paper describes a procedure for solving flexible multibody system on multiple parallel processors with special attention to the computational efficiency. This procedure is improved compared with previous papers by the capability to solve kinematic loops with the similar efficiency as flexible multibody system with kinematic chain or branched kinematic tree. The procedure combines the usage of efficient algorithms on single processor with massive parallelization with the goal to achieve improvement by parallelization even for small systems. The paper describes the extension of the elimination procedure towards the parallelized dynamic solution of flexible multibody system. The method is based on the modified state space and the efficient set of natural coordinates and modal coordinates for the description of the deformation in the system. The elimination process is solved by the efficient combination of elimination process and Cholmod procedure. The resulting computational complexity is very promising. The elimination procedure is applied only to the computation of reaction forces and the local equations of motion are solved completely independently on parallel processors without any constraints between the processors.*

1 Introduction

The solution of the flexible multibody system dynamics can be done using several procedures. They can differ in the type of coordinates being used or in the ability for parallel implementation. There are many effective formalism for which the parallelization is, however, not possible. Usually they are based on the relative coordinates and the recursive computation [7, 8]. A method widely used for the parallel solution of multibody dynamics is the Divide and Conquer (DAC) method [1-3] based on the tree division of the system with systematic reduction of the system size.

The new procedure for solving flexible multibody system on multiple parallel processors with special attention to the computational efficiency was developed in recent years [5, 6, 9, 10] . The procedure combines the usage of efficient algorithms on single processor with massive parallelization with the goal to achieve improvement by parallelization even for small systems. The method is based on the modified state space and the efficient set of natural coordinates and modal coordinates for the description of the deformation in the flexible multibody system. This method uses the natural coordinates for the description of the system position and modal coordinates for the description of the deformation in the system and modified state space approach [4] with Schur complement procedure [5]. The elimination process is solved by the efficient combination of elimination process and Cholmod procedure. The resulting computational complexity is very promising. The elimination procedure is applied only to the computation of reaction forces and the local equations of motion are solved completely independently on parallel processors without any constraints between the processors. The limitation of this approach is that it is valid just for kinematic chain or branched kinematic tree.

The paper describes an extension of this procedure to solve flexible multibody systems containing kinematic loops with the similar efficiency as flexible multibody system just with kinematic chain or branched kinematic tree.

2 Open Loop Procedure

The basic method is based on the modified state space and the efficient set of natural coordinates [8, 4] and modal coordinates for the description of the deformation in the system. The the equation of motion for single body i is derived as

$$M_i \ddot{s}_i + K_i s_i = Q_i \tag{1}$$

where M_i is the mass matrix, K_i is the stiffness matrix and Q_i is the vector of generalized forces. Using the procedure described in [6, 9] exploiting the Schur complement the resulting system of equations of motion (EOM) is obtained

$$M\dot{s} + J^T \mu = p^* \tag{2}$$

$$J\dot{s} = -\alpha f(s) \tag{3}$$

where M is the diagonal mass matrix, J is the Jacobi matrix corresponding to the constraints f, α is the coefficient of the Baumgarte stabilization, s is the vector of natural coordinates describing the absolute system position, p^* is the modified momentum of the system and μ is the vector of the new Lagrange multipliers.

Let us suppose that the multibody system has the structure of kinematic chain (Fig. 1). The structure of the Jacobi matrix J can be simplified into the structure represented by the Fig. 2.

Figure 1: The simple kinematic chain

Figure 2: Structure of the Jacobi matrix

Expressing $\dot{s}$ from (2) and substituing into (3) the resulting system for unknown μ is obtained

$$J M^{-1} J^T \mu = \alpha f(s) + J M^{-1} p^* \tag{4}$$

which can be simply written as follows

$$A\mu = b \tag{5}$$

The system of equations (5) is sparse, symmetric, positive definite with band structure for the case of a simple kinematic chain of n bodies (Fig. 1). The system (4) has a structure of blocks (Fig. 3) corresponding to particular bodies with equivalent (small) sizes.
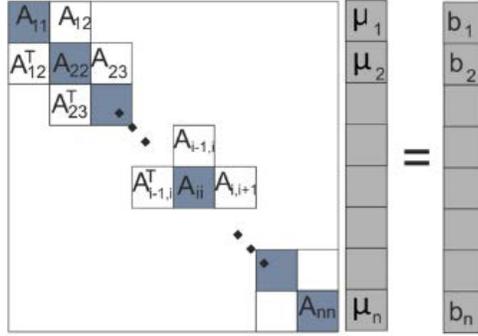
Figure 3: The resulting matrix-structure

Thus the whole system of the equations can be understood as a set of the interconnected subsystems representing by the blocks for unknown vectors $\boldsymbol{\mu_i}$. The number of elimination levels is $log_2(n)$ and this is proportional to the resulting computational costs of elimination process. Based on the comparison of the application of elimination process and Cholesky decomposition the combination of both approaches has been proposed [6, 9]. The result is the efficient combination of elimination process and Cholmod procedure (Fig.2). This combination has been investigated for the small blocks (9x9). In the case, that the 9x9 division is used and there are not enough processors for the matrix transformations, the process is following. The number of subsystems n_s for elimination is the same as the number of bodies n. The number of processors n_p is smaller than n. Therefore it is possible to evaluate only n_p elimination in parallel on one elimination level and the rest has to be carried out after that.
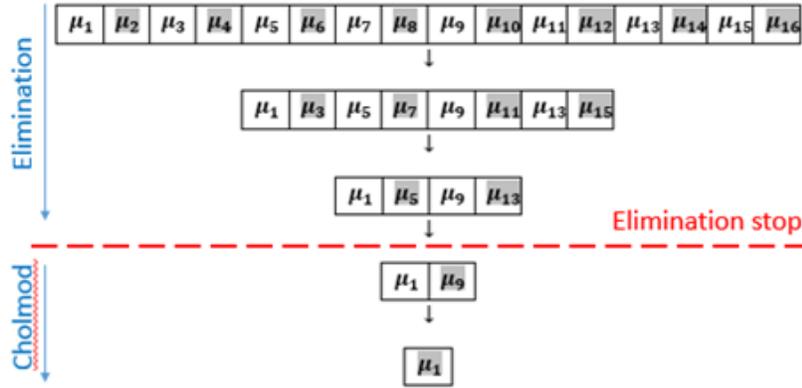

Figure 4: Efficient combination of the elimination with the Cholesky decomposition

It is obvious, that the system of equations (5) can be split into the sub-blocks which number corresponds to the number of processors ($n_s = n_p$). However, it is always better from the complexity point of view to split the system in that way, that the number of sub-blocks corresponds to the number of particular bodies in the kinematical system ($n_s = n$). Thus the optimal elimination process is obtained, see [9, 10]. The complexity of the solution is very promising. The example in [9, 10] demonstrates that 10 times increased of efficiency is possible even on 10 processors for small flexible multibody systems (20 bodies with 10 flexible modes).

3 Closed Loop Procedure

The limitation of this approach is that it is valid just for kinematic chain or branched kinematic tree. The problem is the occurance of kinematic loop that leads to matrix structure from Fig. 3 with broader bandwith. This paper is devoted to the description of solution of this problem.

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The approach for solving the multibody systems with kinematic loops consist in the dividing suitable body within the loop into two bodies and their firm connection. The connection is described by appropriate constraint within the constraints $f(s)$ in (3). The only requirement is that the distance of coordinates describing the position of mutually constrained bodies in the enumeration of coordinates in the vector s is small. This corresponds to the resulting bandwith of matrix in Fig. 3. This can be achieved by repeated division of bodies. The divided bodies has half mass matrix. This leads to the limitation of the body divisions d. Because of reasonable properties of the mass matrix for the numerical integration the reduction of mass of divided bodies to 10-d the number of body divisions should be limited by $d=1-2$, i.e. 10-100 times. This is limitation but it is applicable for smaller flexible multibody systems. This means that the computational complexity of direct dynamic solution for current multibody models of machines can be significantly increased similarly as in [9, 10] for kinematic chains.

The resulting matrix A in (4)-(5) after Schur complement has the bandwith given by the maximum distance of bodies in the corresponding Jacobi matrix (Fig. 2) times the size of one A matrix block (9x9) in Fig. 3.

The division of bodies leads to the increase of bodies and coordinates and potentially the computational costs. However, the computational costs depends on the number of bodies logarithmically and thus the increase of number of bodies is negligible. If the number of bodies is for example increased 2 times then the computational costs are increased just by $log_2(2n)=1+ log_2(n)$.

The example of application of this procedure is in Fig. 5. The simple kinematical loop is in Fig. 5a. Its representation in the order of coordinates is in Fig. 5b. The reduction of distance of constrained coordinates is depicted in Fig. 5c. It is achieved by the dividing the first body in the order into three ones and introducing equivalent constraints. The resulting distance of constrained coordinates is reduced from 6 to 3 and accordingly the bandwith of matrix A.


(a)　　　　　　　　　　(b)　　　　　　　　　　(c)
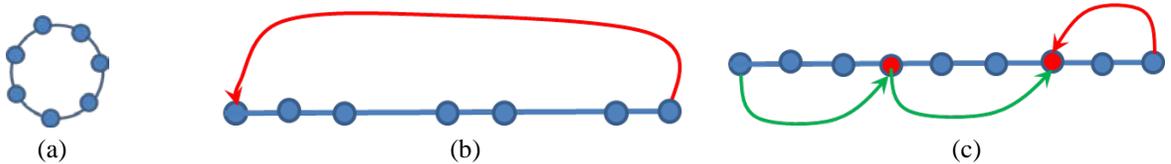
Figure 5: (a) The simple kinematical loop (b) The order of coordinates (c) Solution with added bodies

The structure of the Jacobi matrix J in the structure from Fig. 2 for the example from Fig. 5 is in Fig. 6. Then the bandwith of the resulting matrix A is four times the size of A matrix block (9x9) instead of seven times for original system in Fig. 5a.
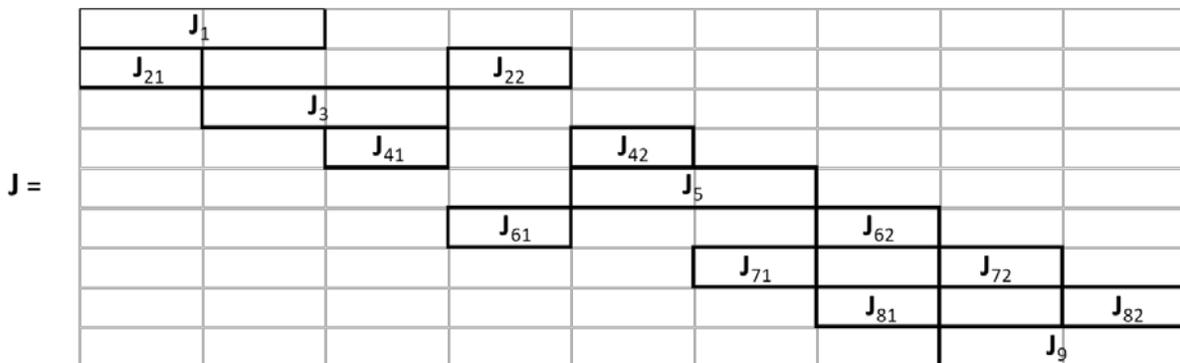
Figure 6: Structure of the Jacobi matrix

4 Conclusions

The paper described the extension of the elimination method of parallelization of flexible multibody system dynamics towards flexible multibody systems with kinematic loops. The computational efficiency remains similar for both open loop and closed loop flexible multibody system. The advantages of original parallelization approach remained. The resulting computational complexity is very promising both for smaller multibody systems of current machines as well as for larger multibody systems.

Acknowledgements

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