

# Dynamic modeling and analysis of contact interaction of a passive biped-walking robot

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**ABSTRACT** — *The main purpose of this work is to present and discuss a general approach for the dynamic modeling and analysis of a passive biped walking, with particular focus on the feet-ground contact interaction. In contrast to McGeer’s passive dynamic models, normal forces and frictional forces acting on the feet and ground have been taken into account in the proposed model. This study aims at examining and comparing several forces models dealing with different approaches in the context of multibody system dynamics. The dynamics equations are obtained by forward dynamics. The normal forces acting on the feet of the passive biped are described based on the viscoelastic contact model. In turn, the frictional forces and the slippage is solve by the equations of Bengisu law for dry friction. For this purpose, a passive walking and the contact and friction force are discussed. The preliminary results obtained are quite promising of the passive biped-walking robot utilized.*

## 1 Introduction

The passive walking is a new concept of biped walking. Researchers have been working on this area with both theoretical and experimental analysis ever since McGeer [1]. Over the decades, many authors have shown that completely unactuated and uncontrolled machines could walk stably downhill on a gentle slope, powered only by gravity, both in numerical simulations and physical experiments.

In fact, different research groups have developed robots based on passive walking techniques [2], namely the one-meter length Robot Ranger of Cornell University [3], with three joints in each of its long legs. Robot Toddlers from MIT University [4] is a small robot that has only a single passive pin joint at the hip and the 3D movement is achieved by means of the feet surface design. The model developed at the Nagoya Institute of Technology has two legs, includes a stability mechanism and is able to walk about 4000 steps (35 minutes) without power supply [5, 6].

Moreover, some actuated prototypes have been constructed based on passive dynamics [7]. The biped “PASIBOT” of Universidad Carlos III de Madrid is able to walk in a steady mode with only one actuator/drive [8, 9]. The robot can walk in a similar way to human, by means of the balance and the dynamic of the natural swinging, in order to consume the minimum energy to walk. This proves that biped robots based on passive walking have good energy efficiency and can perform more natural gaits. It seems that the mechanical parameters of these walkers work better than the complicated control system of the conventional robots in generating natural looking gaits.

Thus, this work deals with a study of a biped-walking robot based on the multibody systems formulation. In a simple way, the planar multibody model considered here includes two legs with spherical feet that can act with ground, which is assumed to be rigid, flat and smooth.

The main purpose of this investigation is to address the supporting foot slippage and contact-impact forces of the biped robot-walking model and to develop its dynamics for simple and double support phases. For this purpose, a general methodology to handle the contact detection between the feet and ground is developed and implemented. Within the spirit of this methodology, special attention is paid to the contact detection itself, both in terms of computational accuracy and efficiency [10]. The normal contact forces developed during the dynamic walking of the robot are evaluated using a Hunt and Crossley based force model that accounts for both elastic and dissipative force components [11].

In turn, the friction forces are computed with different models with the purpose to appraise the most relevant and appropriate options [12]. In the sequel of this process, several parameters associated with the friction forces models utilized here are considered in order to get an appropriate physical and realistic behavior of the passive biped-walking robot.

Then, the dynamic simulations of multibody models used in the context of this work are carried out using general multibody Matlab code named MUBODYNA [13]. This code is able to perform forward dynamic simulations for spatial multibody systems, using several different multibody formulations [14].

## 2 Biped model

Improving a passive walking biped model requires paying special attention to the feet and floor contact. The shape of the feet and the collision of the feet are important in passive walking. To our knowledge, a viscoelastic contact model for the whole walking process has never been used to analyze passive walking. The collisions in most of the models simulated above are rigid plastic collisions (no-slip and no-bounce: non-smooth).

The main aim of our research is to incorporate a more detailed contact into passive walking to work out the viscoelastic contact influence [16]. For this purpose, a general methodology to handle the contact detection between the feet and ground is developed and implemented. Within the spirit of this methodology, special attention is paid to the contact detection itself, both in terms of computational accuracy and efficiency [10].

The viscoelastic contact between the floor and the feet was examined using the Ristow [17] and Schäfer et al.[18] constitutive law (a modified Hertz contact law) [16]. This viscoelastic model was used because Hertz contact model is restricted to frictionless surfaces and perfectly elastic solids. The friction forces are also taken into account. Both feet can slip at any point of the walking process. The Bengisu equation of friction force model [19] is used. Bengisu and Akay proposed a model capable of capturing the Stribeck effect [20].

A multibody dynamic system can be formulated and solved in many ways. In this paper, the equations are implemented in MUBODYNA, a Matlab program [13]. This code is able to perform forward dynamic simulations for spatial multibody systems, using several different multibody formulations [14].

For this algorithm, the relative coordinates are used. Every body has their body-Fixed or local coordinate system, and to describe their rotation the Euler parameters are used. The translational motion is described in terms of Cartesian coordinates, while rotational motion is specified using the technique of Euler parameters.

The kinematic constraints are formulated using generalized coordinates. The constraints equations are associated with kinematic pairs. The program uses Newton-Euler equations of motion, which are augmented with the constraint equations that lead to a system of differential algebraic equations for the motion.

The algorithm integrator utilized in this work in the resolution of the dynamic equations of motion are the Euler method, which is the most popular and uses the numerical integration method, the Runge-Kutta and ordinary differential equations (ode) integrator. The initial conditions are corrected in the program with standard methodology to minimize and avoid the violation constraints.

## 2.1 Geometric considerations of the model

Thus, this work deals with the study of a biped based on the multibody systems formulation on relative coordinates. In a simple way, the planar multibody model considered here includes two legs with round feet that can act with the ground, assuming it to be rigid, flat and smooth. The two legs are linked by a spherical joint. Figure 1, shows the planar model passive dynamic biped on an inclined floor. For simplicity, the model is set as symmetrical, and each leg has a mass of  $m$ , a length of  $l$ , and a moment of inertia of  $J$  about their mass center  $C$ . The round feet have a radius of  $r$  and the centers of the radius are  $O$ . The distance between the mass center of each leg  $C$  and the spherical joint  $S$  is  $d$ .

In the present work, the global coordinate system is denoted by  $xyz$  in which the  $z$ -axis is parallel to the direction of gravity. Every leg has their own fixed local coordinate system of axis. The rotational angles around  $y$ -axis between the leg-fixed or local coordinate system and the global system are denoted by  $\theta_1$  and  $\theta_2$ , respectively. Note that in this case, the  $y$ -axis in global frame is parallel to the  $y$ -axis in local frame and the Euler parameters can be expressed by:

$$p = \{\cos \theta/2, 0, \sin \theta/2, 0\}^T$$

The model consists of a pair of rigid legs, which are named as leg 1 and leg 2, respectively.

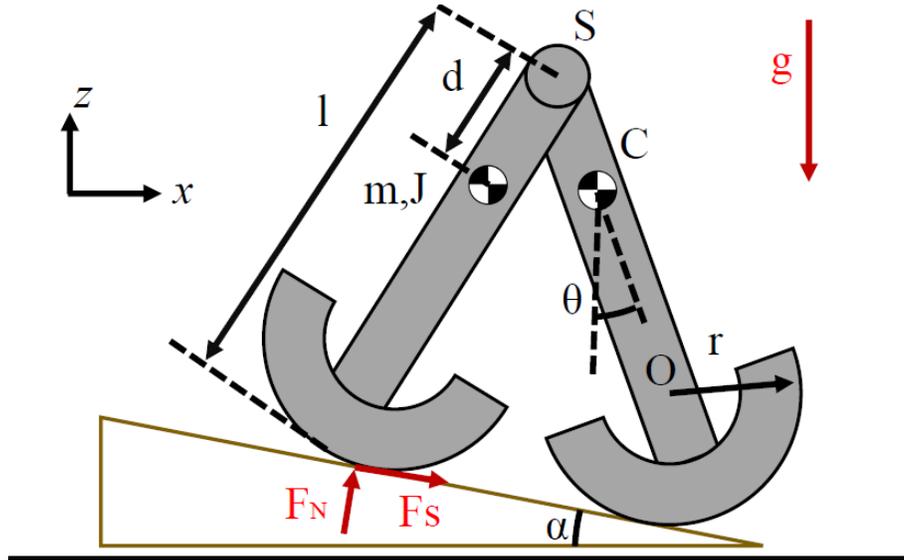


Fig. 1: The planar model of the passive dynamic biped

## 2.2 Normal and Friction force of the model

During the walking of the biped, the feet contact with the floor and small penetrations appear, as shown in Figure 2. The most popular normal contact force model for representing collision is based on the work by Hertz. It should be noted that the Hertz contact theory is restricted to frictionless surfaces and perfectly elastic solids and the energy dissipation during the contact process is not taken into account.

In order to describe the possible energy dissipation, many normal dissipative contact force models are developed. The viscoelastic contact between the floor and the feet was examined using the Ristow [17] and Schäfer et al. [18] constitutive law (a modified Hunt and Crossley Contact Model) with their empirical hysteresis damping factor [11].  $F_N = K\delta^n + c\delta\dot{\delta}$ , where  $F_N$  represents the normal contact force of the contact foot,  $\delta$  is the relative penetration or deformation depth, and  $\dot{\delta}$  denotes the relative normal contact velocity between foot and the floor.  $K$  and  $c$  are the generalized stiffness parameter and the hysteresis factor respectively, which are dependent on the radius of the

feet and the material properties of the feet and floor. In this model the energy dissipation during the contact process has been taken into account.

By using geometrical parameters and generalized coordinates of the biped, the relative penetration depth between foot of one leg and the floor can be obtained.

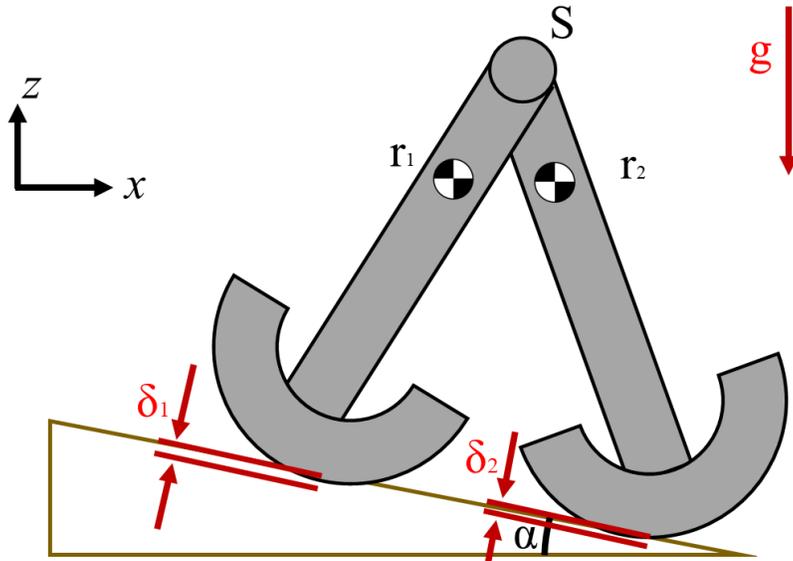


Fig. 2: The contact model of the biped feet with small penetrations

When the stance leg is in contact with the floor, the swing leg may scratch the floor as it swings forward, affecting the normal walking of the passive dynamic walker. In experiments, a special floor in order to prevent the swing leg from scratching the floor was built. In the numerical simulations of this paper, it has been assumed that a swing leg is not subjected to the contact forces to avoid scratching the floor. To prevent the scratching of the swing feet and the ground, the contact will occur only when:  $\delta > 0$  and  $\omega > 0$ .

For the friction model, the majority of models uses Coulomb's dry friction law; however, this model does not take into account the Stribeck effect.

Bengisu and Akay [19] proposed a model capable of capturing the Stribeck effect, as represented in Figure 3. The model is constituted by two equations (one for the slope and another to describe the Stribeck effect).

in which  $\zeta$  should be a positive parameter which represents the negative slope of the sliding state.

$$F = \begin{cases} \frac{F_S}{v_0} ((\|V_T\| - v_0)^2 + F_S) \text{sgn}(V_T) & \text{if } \|V_T\| < v_0 \\ (F_C + (F_S - F_C) e^{-\xi(\|V_T\| - v_0)}) \text{sgn}(V_T) & \text{if } \|V_T\| \geq v_0 \end{cases}$$

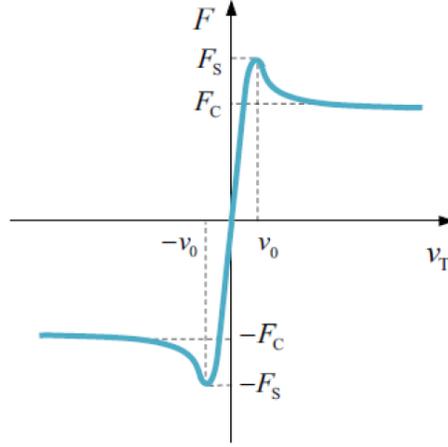


Fig. 3: Representation of the Bengisu and Akay model

These models have the particularities that for low velocities the small time step increment is needed which slow down the simulation, but it has the advantage that for velocities close to zero, the friction force will always be low independently of the displacement. It can describe the characteristics of dry friction and illustrate the stick-slip phenomenon between contacting bodies, and its coefficients can be gained by doing experiments or consulting handbooks on friction. For our model, the coefficients have the values of  $\xi = 1000$  and  $v_0 = 0.0001 \text{ m/s}$ . So the Bengisu and Akay dry friction model is adopted as the friction contact law of the feet and the floor in this paper.

## 2.3 Initial conditions

The initial conditions of the biped have been set from the literature [15]. Thanks to these initial conditions, it is possible to verify the numerical results of the dynamic model with contact and friction forces of this paper with real experimental results.

The parameters of the passive biped are set as:  $m = 1.0 \text{ kg}$ ,  $J = 0.0096 \text{ kg} \cdot \text{m}^2$ ,  $l = 0.40 \text{ m}$ ,  $d = 0.10 \text{ m}$ ,  $r = 0.08 \text{ m}$ .

The gravitational acceleration:  $g = 9.8 \text{ m/s}^2$ .

The exponent of the modified Hertz contact law is set as:  $n = 1.5$

Contact parameters:  $K = 1 \cdot 10^6 \text{ N/m}^{1.5}$ ;  $c = 1 \cdot 10^7 \text{ N} \cdot \text{s/m}^2$ ;  $\mu_0 = 0.50$ ;  $\mu = 0.40$ .

Slop angle:  $\alpha = 0.02 \text{ rad}$ .

The initial conditions of the passive biped:  $\theta_1 = -0.2479 \text{ rad}$ ;  $\omega_1 = -0.0052 \text{ rad/s}$ ;  $\theta_2 = 0.1655 \text{ rad}$ ;  $\omega_2 = -1.2565 \text{ rad/s}$

The spherical joint start at  $(0, 0.395, 0)$ .

The body of the floor is a plane defined by a point  $(0, 0, 0)$  and a normal vector  $(\sin(\alpha), 0, \cos(\alpha))$ . The position of the CoM of leg1:  $r_1 = (d \cdot \sin(\theta_1), 0, -d \cdot \cos(\theta_1))$  and the position of the CoM of leg2:  $r_2 = (-d \cdot \sin(\theta_2), 0, -d \cdot \cos(\theta_2))$ .

The linear velocities of the CoM of the leg1:  $v_1 = (0.37404, 0, 0.037832) \text{ m/s}$  and the linear velocities of the CoM of the leg2:  $v_2 = (0.49847, 0, 0.058404) \text{ m/s}$

### 3 Verification of the model

In order to verify the dynamic model of this paper, the results obtained with the same initial conditions of the literature [15] have been contrasted. It can be seen in Figure 4 that with these initial conditions the biped is walking with stable and passive gait during the slop. This Figure shows one completed gait of the biped model: a set of pictures of the motion produced by the passive biped-walking robot in different phases of the gait. These pictures were obtained with the animation of the MATLAB program. It can be noted that the walking is stable and passive along the slop.

Figure 5 illustrates the phase portraits of one leg relative to a dynamic simulation. It must be said that the response for both legs is the same due the symmetry of the model. In a broad sense, the dynamic behavior of the passive biped-walking robot is consistent with the literature. In particular, Figure 5 indicates that the passive biped gets into a stable walking. The ends of the lower part of the graph are the hit and the lifting of the foot. It can be noted that during the hitting the floor, the swing leg changes into a stance one and its angular velocity changes rapidly. It must be highlighted that the dynamic response of the robot is quite sensitive to the initial conditions and to the values of the contact-impact force parameters.

Figure 6 illustrates the time histories of the leg angle, which is consistent with the result of the literature [15]. The simulation results show that the motions of the two leg have the same stride and period. It can be noted how there are steps in which it gains a little amount of energy (greater angle) or steps in which it loss energy. The simulation results show that the motions of the two legs have the same stride and period.

Figure 7 a shows the time histories of the penetration of the foot and the floor, and figure 8 depicts the time histories of the contact forces of this foot acting on the floor. These calculation results show that foot is subjected to a large impact both in normal and tangential directions when it hits the floor.

In these graphs it can be observed how the contact force increases as the penetration increases, until the point where the other foot impacts, producing a phase of double support. In this phase of double support, an impulse is transmitted from one foot to another, greatly increased contact forces, and producing a rebound that switch the stance leg into swing leg almost instantaneously after the impact.

The contact forces acting on the other leg are identical to that one, except for a certain phase difference. This result is consistent with the literature [15]

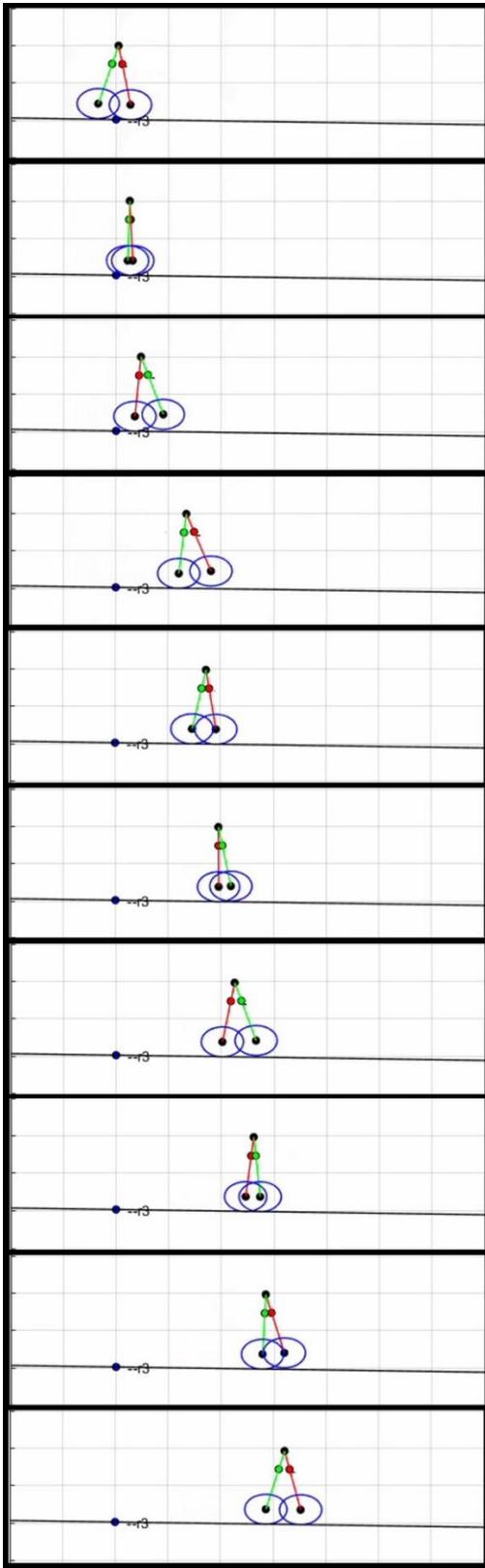


Fig. 4: Passive model gait

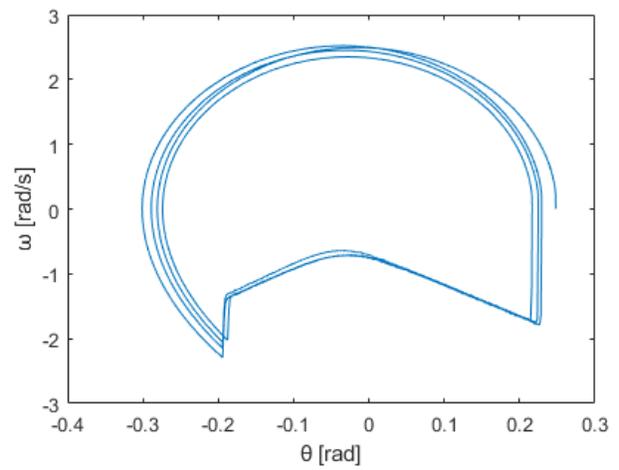


Fig. 5: Leg phase portraits

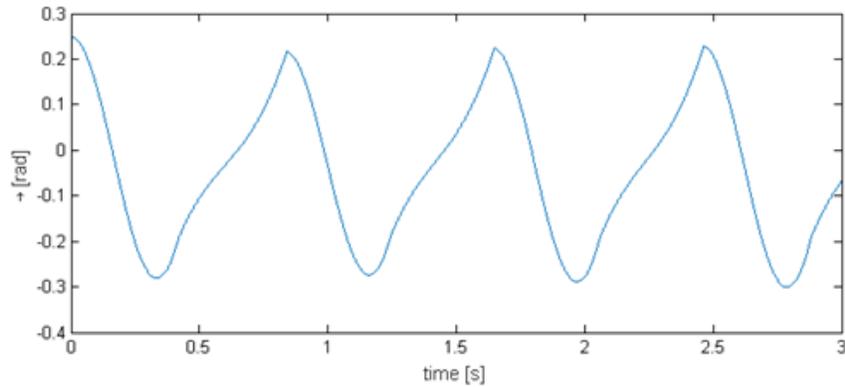


Fig. 6: Time histories of one leg angle

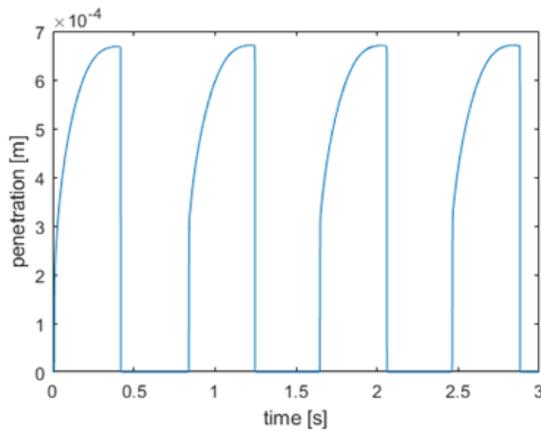


Fig. 7: Time histories of penetration

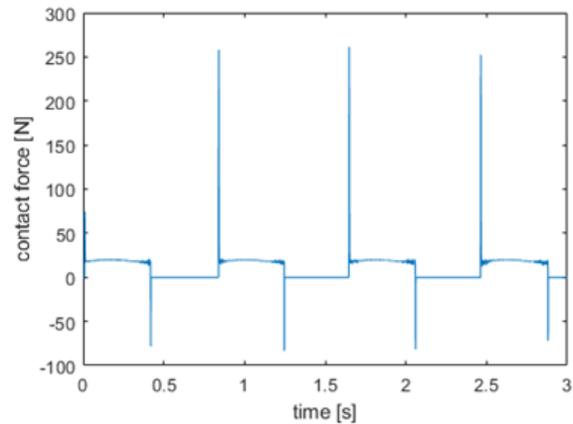


Fig. 8: Time histories of contact forces acting on foot.

## 4 Numerical results

A model with a stable gait that has a large basin of attraction means that this gait can start from points far from the fixed point. In other words, this kind of model is not sensitive to the initial conditions. Thus, models that have large basin of attraction, like the model explained in this paper, might be significant not only in the simulation, but also in the experimental settings. In this section, the most interesting numerical results of the model are shown. First, the effect on the slope has been investigated. The hip speed of the model along the slope varies every time during the walking. In the Figure 9, it can be seen the leg angle during the walking for different slope. It can be noted that for the slope of 0,20 rad, the walking is steady and full passive. For a higher slope, the angle are increasing in every step and gain kinematic energy. This also can observed in Figure 10, where the contact force are plotted for different slopes. Also, for slope higher than 0,20 rad, the contacts force are bigger with every impact. That is to say, that all the gravitational potential energy that is gaining, is greater than the energy that falls on each impact, with which its kinetic energy increases at each step. These results are what we would expect.

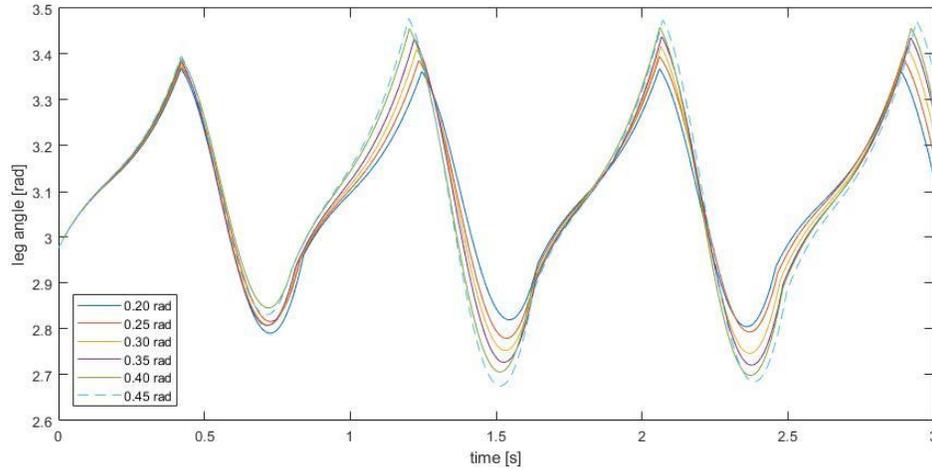


Fig. 9: Leg angle for different slopes

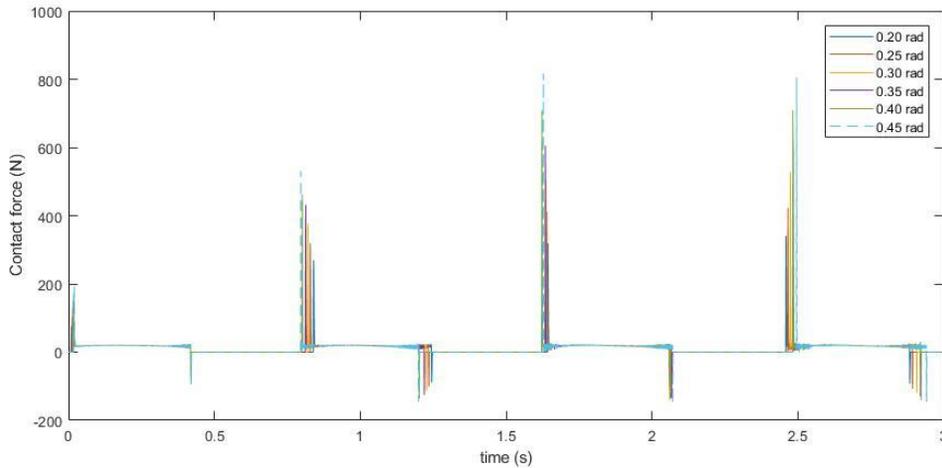


Fig. 10: contact force for different slopes.

The main numerical methods to solve equation of motion commonly used in multibody dynamics system have been applied to the model. Also a comparative study of several methods to handle the constrains violation is presented in this section.

In this particular example, seven methods are utilized to solve the system dynamics, namely the standard Lagrange multipliers method (standard), the baumgarte stabilization approach (baumgarte), the penalty method ( penalty), the augmented lagrangian formulation (augmented), the index-1 projection method (index1), the index-1 augmented lagrangian method (index1aug), and the direct correction method ( described).

For this comparison the differential and algebraic equations of motion of the multibody dynamics system, will be solve with the most popular and used numerical integration methods: First, the ode integrator scheme; second, the euler integrator scheme, and third, the Rung-Kutta methods, are utilized. These methods have been known for more than 100 years, but theis potential was not fully realized until computer became available. These methods involve step-by-step process. All the cases are simulated and analyzed for the same period, and with the same initial conditions.

Figure 11 shows the plots of the constraints violation resulting from the dynamic simulation of the biped model. In this analysis, all the methods named above (standard, baumgarte, augmented, penalty, index1, index1aug, and

described)) are utilized to solve the dynamics system. It should be highlighted that different scales are used for results. From the results shown in Figure 11, as expected, it can be observed that when the standard Lagrange multipliers method is used, the violation of constraints grows indefinitely with time. However, when the other stabilization method is considered, the response is slightly different. In fact, with the baumgarte approach, augmented lagrangian formulation, the index-1 projection method and the direct correction method the constraints violation does not growth with time, instead it tends to stabilize or stay under control, as it can be observed. Furthermore, the augmented lagrangian formulation exhibits better behavior when compared to the previously analyzed approaches. Finally, it can be observed that the direct correction approach completely eliminate the violation of constraints. It should be noticed that constrains violation of the augmented lagrangian formulation, the direct correction approach and the baumgarte stabilization approach have the same order of magnitude and the results are similar.

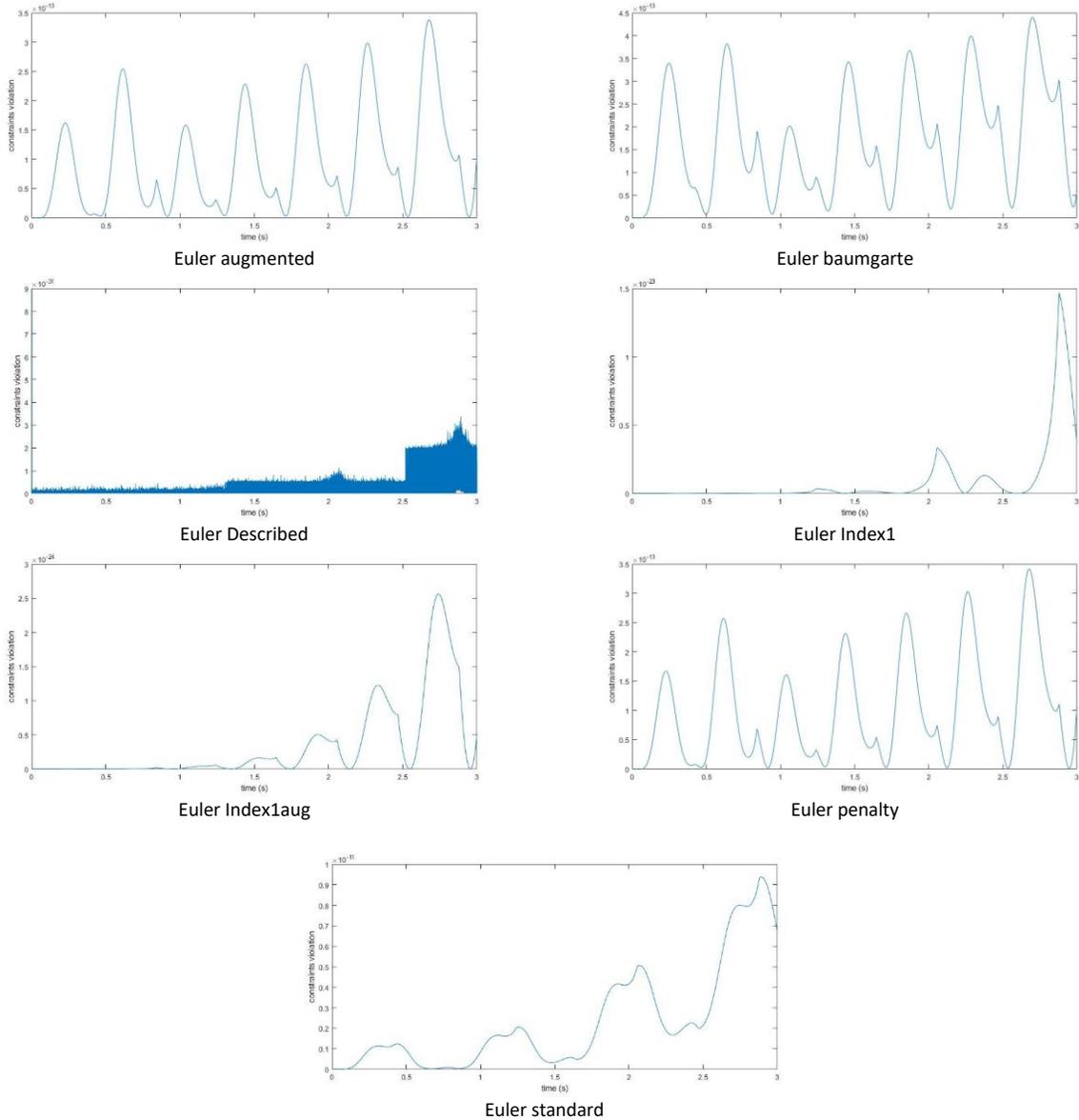


Fig. 11: Position constraints violation of the model

These results are well appreciated in figure 12, where all constraints violations are shown, and in figure 13, where all the methods are shown except the standard one. It should be noted that different scales are utilized for the results plotted in figure 12 and figure 13.

With these results, the robustness of the model has been demonstrated. It work properly with all the methods.

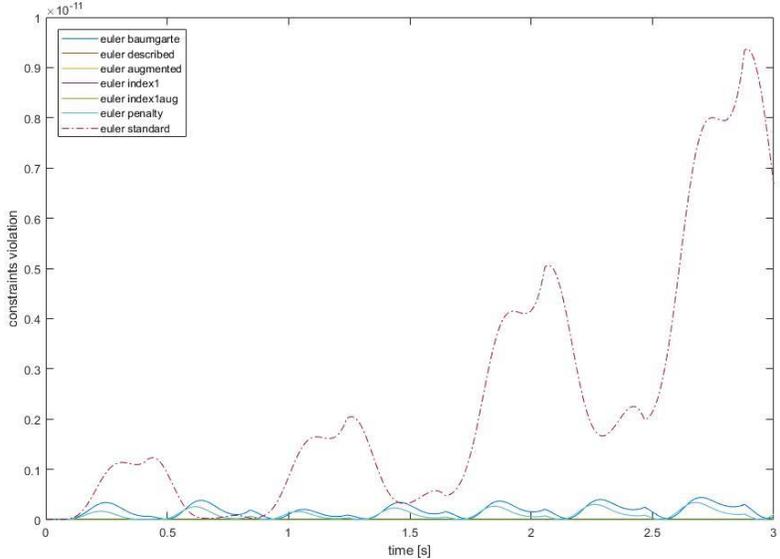


Fig. 12: All the Position constraints violation

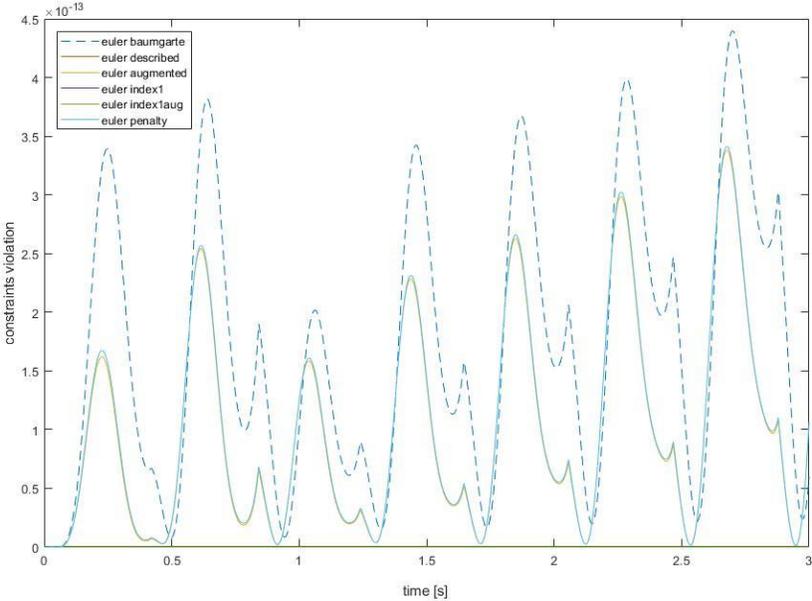


Fig. 13: All the Position constraints violation except standard method

The efficiency of the different methods utilized to solve the dynamics equations of motion of the model is presented in figure 14. As it was expected, the standard method (standard lagrange multiplier) is the most efficient approach. Moreover, the augmented lagrangian formulation and the baumgarte stabilization approach present a similar time ratio. These two method have much less constraints violation when compared with the standard one.

For this reasons, the best approaches in terms of computational eddiciency for this particular model are the standard method, baumgarte method and augmented method, as it can be seen in the chart of figure 14.

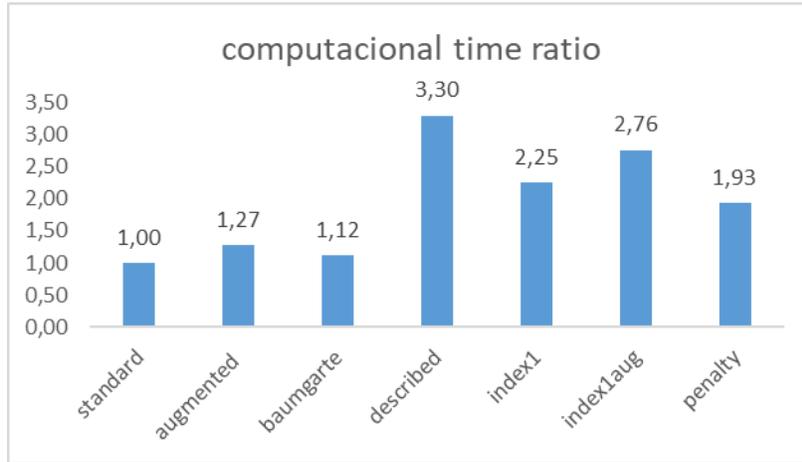


Fig. 14: Computation time ratio for the different methods

In the previous paragraph, the equation of motion for the dynamics model were derived and integrated with the Euler integration method. As seen in the results, the augmented method have under control the constraints violation and the efficiency is similar to the standard methods. Then, the integration method have been compared using the augmented method and the standard. In Figure 15, the ordinary differential equations, “ode”, integrator, the “euler” and the Runge-Kutta, “runge2”, are compared using the augmented method. It should be noticed that the three graphs have very different order of magnitude, and that the method that most eliminates the constraints violations is the Runge-Kutta

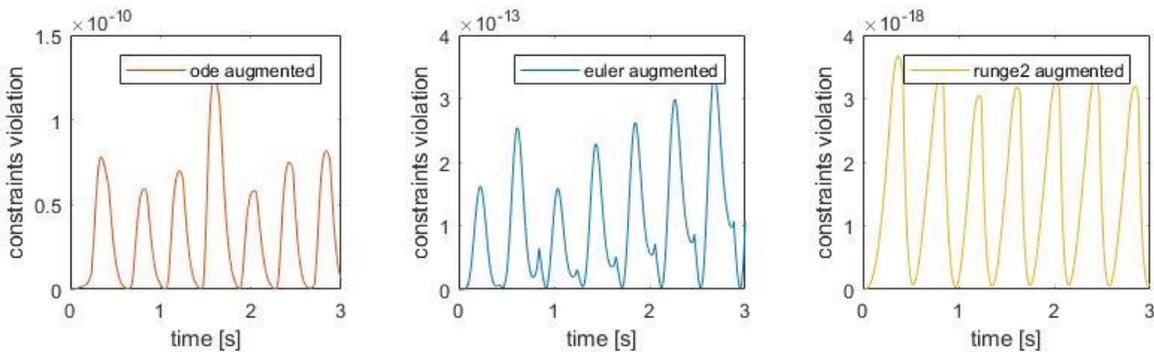


Fig. 15: Computation of the constraints violation for the different integrator (augmented method)

Also, the same comparison has been made with the standard method, achieving similar results. This is shown in the figure 16, where it is easy to check that Runge-Kutta integrator controls better the constraints violation. It should be noticed that the three graphs have very different order of magnitude.

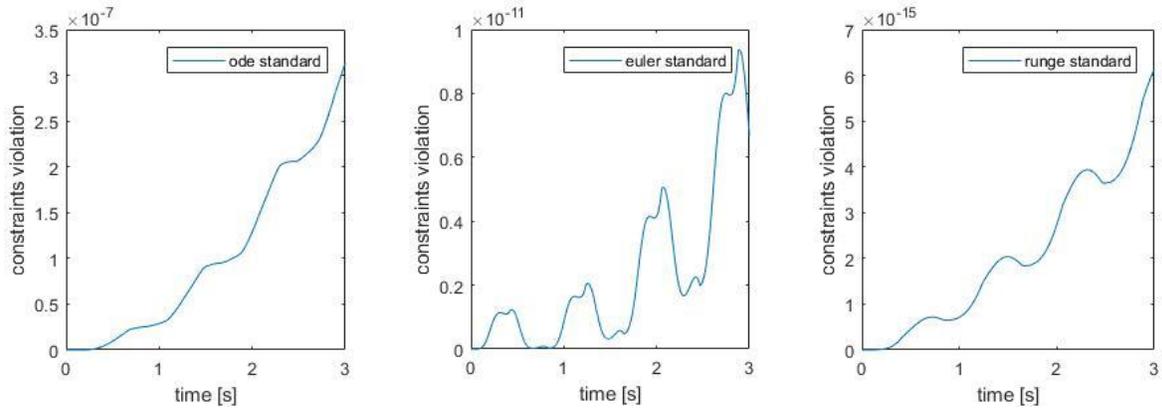


Fig. 16: Computation of the constraints violation for the different integrator (standard method)

The efficiency of the different integrator utilized to solve the dynamics model are presented in figure 17. As it was expected, the Runge-Kutta integrator (with both methods: standard and augmented) is the less efficient approach. It needs more operations to control the constraints violations. Moreover, the ode integrator needs much less time than the Runge-kutta integrator and the Euler integrator. With these results, it can be concluded that for this model, the augmented method with the ode integrator is a good combination, keeping the constraints violation under an intimate value and not needing too much time for it.

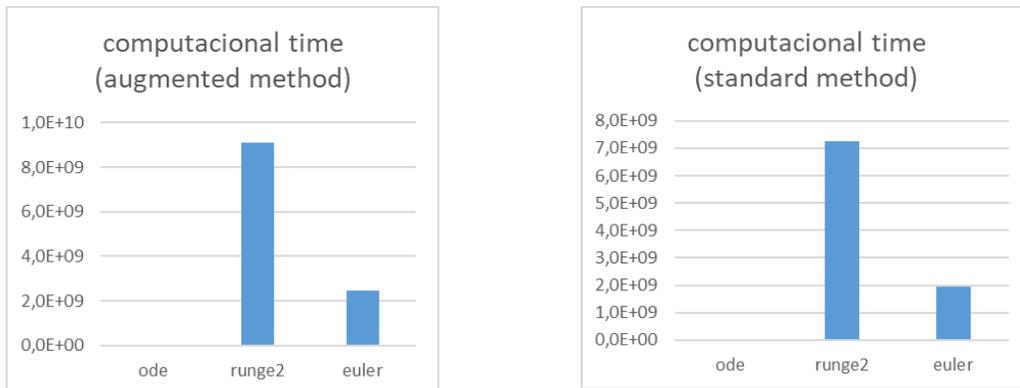


Fig. 17: Computation time for the different integrators for augmented and standard method

## 5 Conclusions

A planar dynamic model for the whole passive walking of a biped robot-walking with the supporting foot slippage and contact-impact forces has been presented. The results have shown that the contact force and friction force equations work properly for this biped model. These results have been contrasted with the literature. For these reasons, it can be concluded that the model with contact/impact between the passive foot and the floor works correctly.

Knowing that the model works properly, a sensitivity analysis has been carried out, obtaining interesting results with different numerical methods to solve equation of motion (Lagrange multipliers method, the baumgarte stabilization approach, the penalty method, the augmented lagrangian formulation, the index-1 projection method, the index-1 augmented lagrangian method, and the direct correction method) and different integrator schemes (ode integrator scheme; euler integrator scheme, and the Rung-Kutta methods). Those results have been shown and are consistent. With these results, it can be concluded that for this model, the augmented method or baumgarte method with the ode integrator is a good combination, keeping the constraints violation under an intimate value and not needing too much time for it.

The model is implementer in a parametric programs, which make possible the option to do the sensitive analysis.

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