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Multibody based topology optimization including manufacturing constraints

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ABSTRACT — *Over the past decades topology optimization has gained large interest in structural design since it allows fast development of optimized complex geometries. These conceptual designs will often lead to superior properties compared to designs developed using classical human design techniques. However, when this optimization is performed for components which are part of a multibody system (MBS), design changes on the body can have a significant impact on dynamic behavior of the system and its loading conditions. This interaction therefor needs to be taken into account in the optimization.*

In this work we propose a fully coupled flexible multibody component structural topology optimization approach, where the design objective is directly evaluated and improved from the flexible multibody response. This is in contrast to classical methods where the MBS model only serves to generate loads which are applied to a separate finite element model.

By applying a novel efficient flexible multibody formulation, based on the reduced small deformation component response, the entire optimization workflow can be made sufficiently efficient for practical application on large meshes. Moreover an extrusion constraint is added to the optimal design problem in order to further limit the computational load by limiting the design space, and to obtain practically cheap-to-manufacture designs.

The proposed methodology is demonstrated on the structural topology optimization of the connecting rod in a slider crank. The proposed approach enables fast convergence to a practically manufacturable design.

1 Introduction

Over the past decades structural topology optimization has gained large interest in structural design as it enables fast development of optimized complex geometries. These designs will often lead to superior properties compared to classical human designs [1, 2, 3].

Although topology optimization was originally developed for optimizing single components rather than full mechanical system assemblies, several methods have been proposed in literature to account for the dynamic behavior of multibody systems. Most notably Kang proposed the method of Equivalent Static Loads (ESL)[4], which remains the basis method for these applications.

The ESL approach transforms the dynamic loads obtained from a multibody simulation into a static optimization problem consisting of multiple critical load cases for a single body. In this weakly coupled approach the optimizer does not explicitly take the multibody interaction into account, but only re-evaluates the component loads from the multibody model after a full topology optimization run on the statically loaded component.

This decoupling is convenient as it enables the exploitation of of-the-shelf multibody software and structural topology optimizers, but also has several drawbacks: the ESL aproachs for example makes it particularly challenging to introduce constraints directly based on the dynamic response of the component.

In this work we therefore propose a fully-coupled approach for component topology optimization in a (flexible) multibody system (MBS). In this approach, a full flexible multibody simulation is performed for each topology iteration. The design goal and constraints are then evaluated on the body response from this MBS simulation. This resolves the issue of imposing artificial constraints to the body to eliminate the rigid body motion as required for ESL based approaches [5].

The computation of the response derivatives with respect to the topology parameters, as required for the optimization iterations, is enabled through the use of a recently proposed flexible multibody formulation, namely the *flexible natural coordinate formulation* (FNCF) [6].

Moreover, in order to ensure feasible designs from a manufacturing perspective, we include a manufacturing constraint in the optimization process. Specifically we optimize components with the addition of an extrusion constraint, as extrusion offers a particularly cost-effective production scheme.

The overall topology optimization scheme is discussed in Sec. 2 and a numerical example is provided in Sec. 3.

2 Optimization Framework

This work focuses on the design of mechanisms for industrial mechatronic applications. Often these applications require lightweight components in order to achieve the higher operational speeds without compromising the normal operation. Typically the flexible deformation is assumed to be small in comparison to rigid motion of the components. However it can have an important impact on the functional behavior of the machine.

Fortunately these deformations are sufficiently small in practice such that a small-deformation flexible multibody model can be used, rather than a fully geometrically nonlinear finite element model. The latter would lead to unacceptable computational loads on a full system level optimization for most practical applications.

This section presents an approach for component structural topology optimization based on flexible multibody responses. First the overall scheme is discussed in Sec. 2.1, the specific FMBS approach used is briefly summarized in Sec. 2.3, the optimizer is discussed in Sec. 2.4, and a specific approach to obtain extrusion profiles is discussed in Sec. 2.5.

2.1 Fully-coupled topology optimization

The overall scheme for the structural topology of a component in a flexible multibody system proposed in this work, is summarized in Fig. 1.

The initial topology parameters $\mathbf{x}$ represent the available design space for the novel component, bounded by the physical space available in the machine. These parameters are connected to the *density* of the elements in a Finite Element (FE) discretization of the design space [1]. With these parameters and component FE model, there is an associated multibody model $\mathscr{M}(\mathbf{x})$ which is simulated to evaluate the system and body response $\mathbf{q}$. The topology parameterized body description and associated FMBS model are discussed in Sec. 2.2-2.3.

The corresponding flexible response $\mathbf{q_F}$ is then used for evaluating the design objective $\varphi(\mathbf{q_F}, \mathbf{x})$ and design constraints $\boldsymbol{\gamma}(\mathbf{q_F}, \mathbf{x})$. Next the topology design update $\Delta\mathbf{x}$ is computed and the convergence is evaluated on the magnitude of this update (the optimizer stops if this update is smaller than a prescribed tolerance *tol*). The optimizer and constraints are discussed in Sec. 2.4-2.5.

2.2 Topology parameterized flexible body description

For small deformation problems, as discussed here, the global finite element (lumped) mass $\mathbf{M_{FE}}$ and stiffness $\mathbf{K_{FE}}$ matrices can be described as a function of the *density* parameter x_i for element i:

$$\mathbf{M_{FE}} = \sum_{i=1}^{n_e} \mathbf{M^0_{FE,i}}\, g(x_i), \tag{1}$$

$$\mathbf{K_{FE}} = \sum_{i=1}^{n_e} \mathbf{K^0_{FE,i}}\, g(x_i). \tag{2}$$

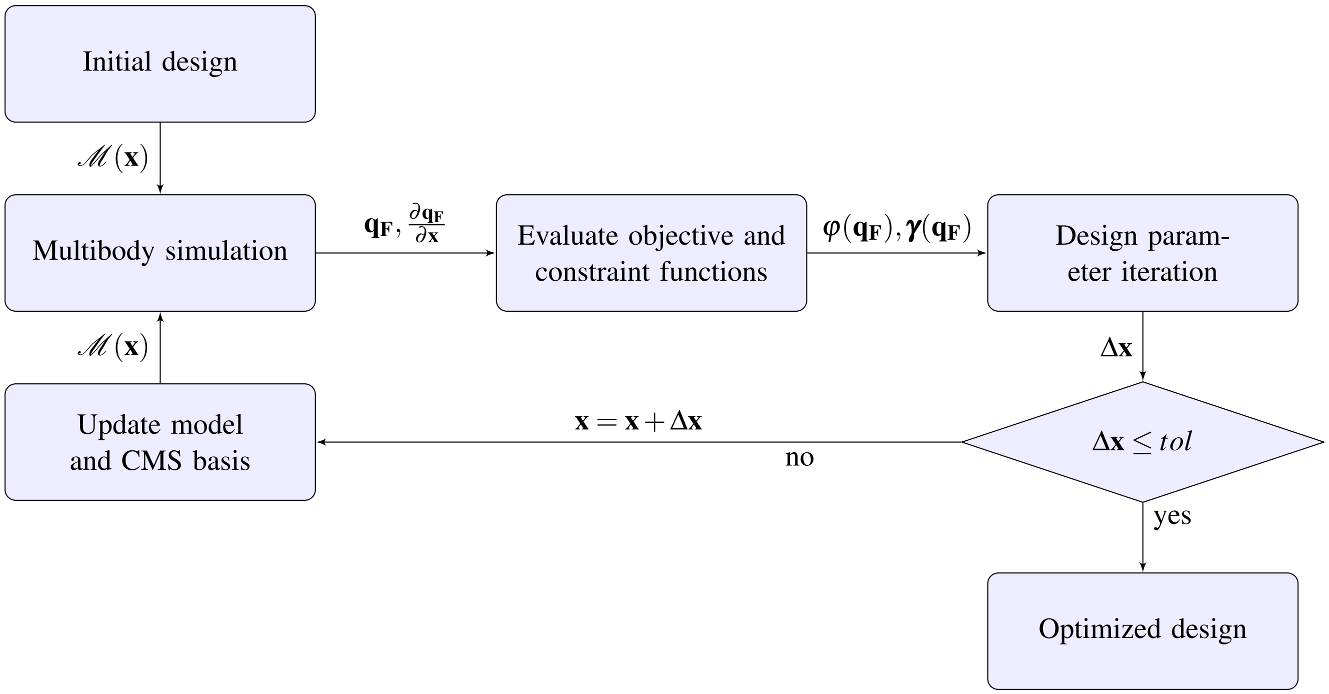

Fig. 1: Schematic of the steps for multibody based component structural topology optimization

Here $\mathbf{M^0_{FE,i}}$ and $\mathbf{K^0_{FE,i}}$ represent the contribution to the full body mass and stiffness matrix from element i for the full design domain finite element discretization and $g(x)$ is a scalar function to describe the dependency on the density parameter. The parameter x_i then indicates how much each element contributes to the optimized model and this parameter should be as close as possible to either 1 or 0.

A fundamental issue when including a finite element model in a multibody model is the large model size. The small deformation assumption partially eliminates this issue as it allows the use of linear model order reduction (MOR) techniques. In this work, we exploit the well-known Component Mode Synthesis technique to approximate the local displacement field of the FE model $\mathbf{u} \in \mathbb{R}^{n_{DOF}}$ by projecting an appropriate set of n_m assumed modes gathered in $\mathbf{\Psi} \in \mathbb{R}^{n_{DOF} \times n_m}$ such that

$$\mathbf{u} \cong \mathbf{u_0} + \mathbf{\Psi}\mathbf{q_F} \tag{3}$$

where $n_m \ll n_{DOF}$. Due to the MOR-approach the computational cost of the MBS simulation is not affected by the number of elements but rather by the amount of assumed modes. This reduction basis $\mathbf{\Psi}$ is computed from the body mass and stiffness matrices and is therefore also a function of the topology parameters $\mathbf{x}$. It is important to highlight that this implies that the reduction basis needs to be recomputed for every topology parameter update, which can also lead to considerable computational loads for large scale problems.

The original nodal mesh positions $\mathbf{u_0}$, reduction basis $\mathbf{\Psi}$ and mass and stiffness matrices are the body information which is passed to set up the multibody model, as discussed in the next section.

2.3 Flexible Natural Coordinate formulation for system and body dynamics

To model the flexible multibody dynamics, we exploit the Flexible Natural Coordinate Formulation (FNCF) as originally suggested by Vermaut [6]. This FNCF approach allows a fast and easy evaluation of the dynamic multibody simulation which is important due to the iterative nature of optimization routines. The formulation is based on a small deformation assumption. A particularly desireable property of this approach is the simple structure of the resulting equations of motion: the dynamic balance equations reduce to a set of (bi)linear equations and the constraint equations are a set of quadratic equations. Here, we briefly summarize the linear structure of the dynamic balance in the equations of motion. For the full derivation of the formulation we refer the reader to Vermaut *et al.* [6].

The generalized multibody coordinates $\mathbf{q}$ are

$$\mathbf{q} = \begin{bmatrix} \mathbf{q_P} \\ \mathbf{q_R} \\ \mathbf{q_G} \\ \mathbf{q_F} \end{bmatrix}, \tag{4}$$

which holds a set of displacement $\mathbf{q_P}$ and redundant orientation $\mathbf{q_R}$ coordinates for each body followed by a set of redundant modal participation factors expressed in the inertial $\mathbf{q_G}$ and body attached $\mathbf{q_F}$ reference frame. It is important to note that a set of intra-body constraints are necessary due to the redundant flexible description:

$$\mathbf{q_G} = \mathbf{q_F} \otimes \mathbf{q_R} \tag{5}$$

where $\otimes$ represented the Kronecker product.

The equations of motion are derived from the Lagrangian which yields

$$\mathscr{L}(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\lambda}) = E_{kin}(\mathbf{q}, \dot{\mathbf{q}}) - E_{ela}(\mathbf{q}) - \boldsymbol{\lambda}^T \boldsymbol{\phi}(\mathbf{q}) \tag{6}$$

where $\boldsymbol{\phi}$ holds a set of intra- and inter-body constraint equations accompanied by their Lagrangian multipliers $\boldsymbol{\lambda}$. The equation of motion are then obtained by applying Hamilton's principle to the Lagrangian:

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathscr{L}(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\lambda})}{\partial \dot{\mathbf{q}}}\right) - \frac{\partial \mathscr{L}(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\lambda})}{\partial \mathbf{q}} = 0 \\ -\frac{\partial \mathscr{L}(\mathbf{q},\dot{\mathbf{q}},\boldsymbol{\lambda})}{\partial \boldsymbol{\lambda}} = 0 \end{cases} \tag{7}$$

This leads to the following set of equations of motion in the generalized coordinates $\mathbf{q}$:

$$\begin{cases} \mathbf{M}_{qq}(\ddot{\mathbf{q}}) + \mathbf{K}_{qq}\mathbf{q}) + \frac{\partial \boldsymbol{\phi}^T}{\partial q}\boldsymbol{\lambda} = \mathbf{f}_{ext}(\mathbf{q}, \dot{\mathbf{q}}, t) \\ \boldsymbol{\phi}(\mathbf{q}) = 0 \end{cases} \tag{8}$$

where an optional external force term ($\mathbf{f}_{ext}$) has been included.

For the FNCF description, in contrast to the classical floating-frame-of-reference (FFR) approach, the generalized mass matrix is configuration independent and has the following structure for a given body with nodal mesh coordinates $\mathbf{u_0}$ and reduction basis $\boldsymbol{\Psi}$:

$$\mathbf{M}_{qq} = \begin{bmatrix} \begin{bmatrix} \sum_{i=1}^{N} m_i & \bar{u}_0\vec{m} & \bar{\Psi}\vec{m} \\ & \bar{u}_0\tilde{M}\bar{u}_0^T & \bar{\Psi}\tilde{M}\bar{u}_0^T \\ symm & & \bar{\Psi}\tilde{M}\bar{\Psi}^T \end{bmatrix} \otimes I_3 & 0_{(12+9n_m)\times n_m} \\ 0_{n_m\times(12+9n_m)} & 0_{n_m\times n_m} \end{bmatrix} \tag{9}$$

where $\tilde{M}$ is the lumped diagonal mass matrix for the FE body, $\vec{m}$ is a vector containing all the lumped mass values of the finite element model, m_i is the i-th lumped mass contribution. The operator $\bar{\cdot}$ represent the split of a vector along the nine components of a rotation matrix for obtaining the global contribution of local components under any rotation [7, 6]. It is important to note that a mass matrix consisting of only uncoupled translational components is invariant for rotation, as has been discussed by Gerstmayr & Ambrosio [7]. The resulting mass matrix for the FNCF approach is therefore also constant.

Under the proposed FNCF approach, the generalized body stiffness matrix becomes:

$$\mathbf{K}_{qq} = \begin{bmatrix} 0_{(12+9n_m)\times n_m} \\ \Psi^T \end{bmatrix} K_{FE} \begin{bmatrix} 0_{n_m\times(12+9n_m)} & \Psi \end{bmatrix} \tag{10}$$

Note from the brief discussion of FNCF that the method has both a constant mass and stiffness matrix at the cost of a larger set of generalized coordinates in comparison to the FFR approach. Furthermore if viscous damping, such as Rayleigh or modal damping, is assumed the resulting damping matrix will also be constant. This structure contributes to the computational efficiency of the FNCF methodology and makes it particularly easy to obtain derivatives with respect to the topology design parameters.

2.4 Optimization scheme

Starting from the FNCF multibody formulation presented above, the general topology optimization problem for the *density* parameters $\mathbf{x}$ of a body can be summarized as:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \varphi(\mathbf{x}) \\ \text{s.t.} \quad & \begin{cases} \mathbf{M_{qq}}\ddot{\mathbf{q}} + \mathbf{K_{qq}}\mathbf{q} + \frac{\partial \boldsymbol{\phi}^T}{\partial \mathbf{q}}\boldsymbol{\lambda} - \mathbf{f}_{ext} = \mathbf{0} \\ \boldsymbol{\phi}(\mathbf{q}) = \mathbf{0} \\ x_{min} \leq x_i \leq x_{max},\ i = 1, \ldots, n_e \\ \gamma_j(x) \leq \gamma_{jmax}, \quad j = 1, \ldots, n_{con} \end{cases} \end{aligned} \tag{11}$$

where n_e is the number of design variables and n_{con} is the number of optimization constraints. The first two constraint equations account for the dynamic behavior of the multibody system. The third constraint equatation represents the bounds for the i-th density variable x_i and the last set of constraint equations can include general design constraints like the maximum deviation of a prescribed trajectory, a maximum value for Von-Misses stresses or many other limitations.

For this topology optimization approach with *density* parameters, the problem is essentially a binary programming problem: the design variables should either be 0 or 1 indicating that the domain represented by the element is a void or solid material. In order to avoid expensive combinatorial searches, the binary problem is relaxed in the sense that intermediate values are allowed during optimization. However, in order to prevent physically unsensible designs, the parameters are penalized such that when convergence is reached a close to *black-and-white design* is obtained.

To asses the convergence of the proposed design we use the so called grayness value G [8, 9]. This is a scalar value indicating the discreteness of the design. If G equals 1 this indicates a fully gray design meaning that all elements i have $x_i = 0.5$. On the other hand if $G = 0$, it indicates a fully black-and-white design, meaning that all elements have either $x = 0$ or $x = 1$. Hence near-zero values of G are an indication of a converged design:

$$G(x) = \frac{4}{n_e}\sum_{i=1}^{n_e} x_i(1 - x_i) \tag{12}$$

In this work, we utilize the solid isotropic material with penalization method (SIMP) as suggested by Sigmund to penalize the intermediate density values in order to obtain a near black-and-white solution [1]. With this SIMP approach the material properties are expressed as:

$$E_i(x) = x_i^p E_0 \tag{13}$$

where E_0 is the Young's Modulus of the isotropic material and $p \leq 1$ is the penalization power. If the penalization factor is larger then 1, the intermediate design variables are penalized. The full body stiffness matrix can then be found from:

$$\mathbf{K}_{FE}(x) = \sum_{i=1}^{n_e} x_i^p \mathbf{K}_{FE,i}^0, \tag{14}$$

where $\mathbf{K}_{FE,i}^0$ is the initial element stiffness matrix assembled with E_0, and which fits the framework presented in Sec. 2.2.

Several optimization routines can be used to solve equation (11). Both heuristic schemes and numerical programming methods are readily available from literature. Exampes are found in: ConLin [10], GCM [11], Sequential Quadratic Programming (SQP) [12], Surrogate Based Optimization (SBO) [13] and Method of Moving Asymptotes (MMA) [14]. The selection of a suitable optimizer depends on the characteristics of the problem and the available information e.g. existence of gradient.

2.5 Manufacturing constraints: Extrusion constraints

Inherent to topology based optimization is the freedom of the optimizer to propose structures which are ideal for conceptual design but can be difficult to produce using classical low-cost fabrication processes. This can be circumvented by introducing manufacturing constraints. Among the many fabrication processes extrusion is a low cost production process which is used whenever components of constant cross section can be applied. In this work we include an extrusion constraint.

Since extrusion assumes a constant cross-section for the entire component the original 3D problem can be simplified to finding the optimal material distribution in a 2D space. This is done by mapping the design variables of the individual 3D elements onto a corresponding projected plane. The resulting problem has much less variables.

The proposed cross-sectional projection is a heuristic scheme which consists of the following steps:

1. determining the centroid of each element of the reference 3D mesh
2. projecting the centroids on the 2D space
3. clustering the projected centroids and construct an appropriate mapping

Several methodologies can be used to determine the centroid of the element, we utilize the shape functions of the element such that the centroid is trivially found via

$$u_c = \sum_{i=1}^{n_k} N_i(0,0,0)\tilde{u}_i \tag{15}$$

where N_i is the shape function for the i-th node of the current element and $\tilde{u}_i$ are the global coordinates of that node.

For each centroid we perform a projection to determine it's local ξ and η coordinates of the 2D space. The projection plane is assumed to be orthogonal to the extrusion curve which is taken at the neutral axis of the design space. The concept of the clustering is depicted in Fig. 2.

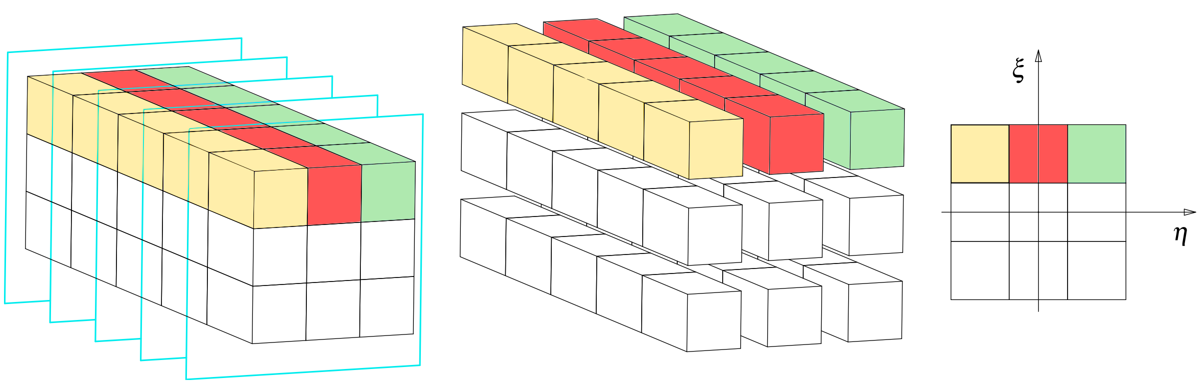

Fig. 2: Clustering of the design variables

3 Numerical example

3.1 Problem description

The considered model is a slider crank mechanism based on the example presented in Moghadasi *et al.* [9]. The example considers the topology optimization of the connecting rod which is discretized in a rectangular domain with 120000 hexahedral elements. The input crank angle is a smooth linear function starting at $\theta(0) = 0rad$ and increasing to $4\pi rad$ at $t = 2s$.

The design problem is formulated as a minimization of compliance, this is a typical objective function for structural topology optimization [1, 15, 9]. The objective function for a body with parameterized stiffness matrix $\mathbf{K}_{FE}(\mathbf{x})$ takes the form:

$$\varphi(\mathbf{x}) = \mathbf{q}_F^T \mathbf{\Psi}^T \mathbf{K}_{FE}(\mathbf{x}) \mathbf{\Psi} \mathbf{q}_F \tag{16}$$

where $\mathbf{q}_F$ are the local flexible participation factors obtained from the FNCF model. For this case the connecting rod flexible deformation in the multibody model is described using 16 CMS modes. The design space is limited by a constraint on the allowable amount of volume, in this example we used a volume fraction of 50%. The topology optimization is performed both with and without the extrusion constraint.

3.2 Results

This section describes the results obtained for the structural topology optimization of the connecting rod in the above described slider-crank system.

Fig. 3 shows the optimization domain and optimized geometry of the connecting rod with and without the extrusion constraint. Without the extrusion constraint, although it is not easily visible on a 2D picture, we obtain a

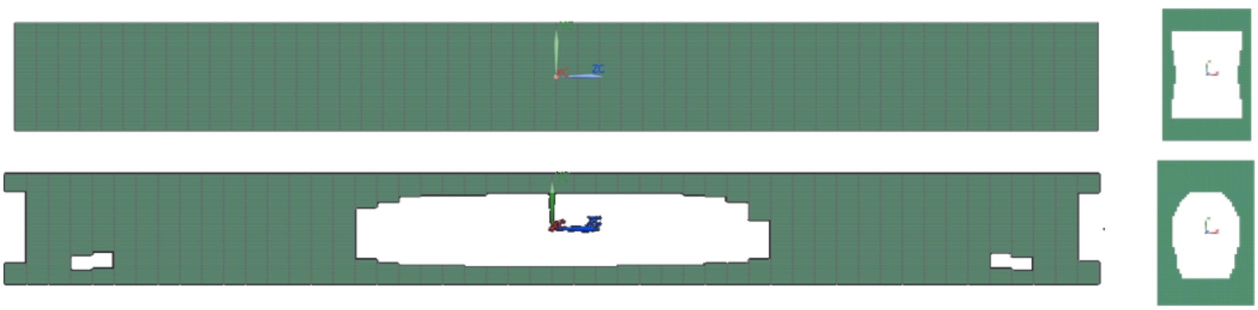

Fig. 3: Top figures show optimized topology with extrusion constraint, the bottom figures depict the optimized topology without extrusion constraint.

typical type of geometry which is expected from these types of topology optimization. Obviously this would be a rather expensive geometry to manufacture and even though it might have highly desirable properties, this type of geometries will see little practical application. In practice this type of design will require post-processing to ensure manufacturability at a reasonable price. The result obtained with the extrusion constraint on the other hand would be low cost to manufacture, but still show a sufficiently interesting geometry to warrant the use of an automatic optimization tool.

The application of the extrusion constraint has only a minor impact on the final performance of the proposed design in this example. Fig. 4 shows the deviation of the motion of the piston mass compared to a perfectly rigid system for both geometries. By applying the proposed multibody topology optimization scheme, the effect of different design constraints on the resulting optimal geometry can be easily evaluated. If for example motion accuracy is more important, the volume fraction constraint could be relaxed and augmented, or replaced, by a tracking accuracy constraint.

Finally Fig. 5 shows the fully coupled topology optimization convergence behavior over the different optimizer iterations for the connecting rod with the extrusion constraint. This figure shows the rapid convergence to a final design. As the applied extrusion constraints strongly limit the search space for the optimizer, only a very limited number of iterations is required for the proposed approach. This low number of iterations coupled with an efficient flexible multibody simulator leads to low overall optimization costs for the proposed framework. The grayness evaluation indicates that a practical design is obtained, whereas the compliance clearly demonstrate how the design stiffens over the different iterations. The norm of the parameter change varies closely with these two other measures and is used in practice to terminate the topology optimizer.

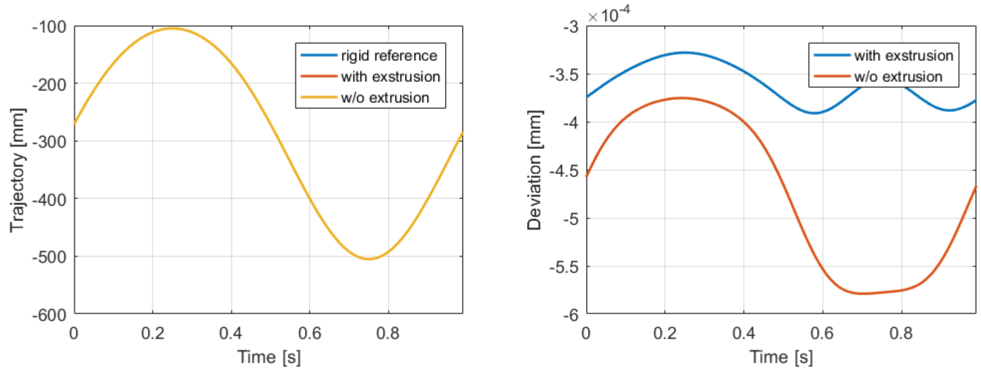

Fig. 4: Piston mass trajectory and trajectory deviation with respect to rigid reference.

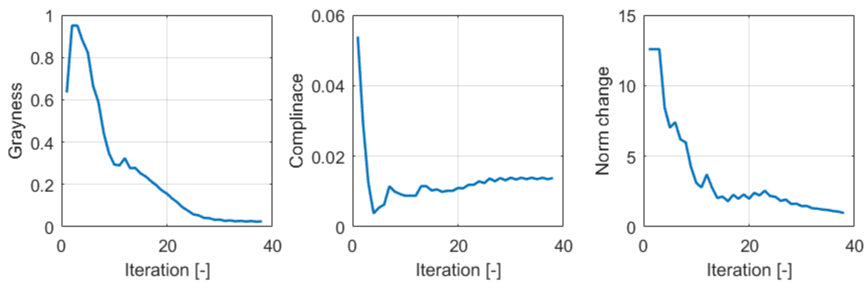

Fig. 5: Convergence over iterations for extrusion constrained coupled multibody topology optimization of slider-crank

4 Conclusions

This work has shown a fully coupled multibody based component structural topology optimization approach. This is in contrast to previous work in literature where a multibody simulation was run to extract representative loads for a separate body topology optimization based on a linear finite element model. In the proposed approach the flexible participation factors obtained from the flexible multibody simulation (FMBS) are directly exploited to evaluate typical topology optimization goal functions, like elastic potential energy over time. In order to enable an overall sufficiently efficient framework, this work uses the recently proposed flexible natural coordinate formulation (FNCF). To further speed up the optimization and obtain more feasible design for manufacturing, the authors also exploited an extrusion constraint. The proposed approach is demonstrated on an optimal design for a connecting rod in a slider-crank mechanism. This example shows rapid convergence for a practically applicable design. In the current framework, one of the main bottlenecks is the recomputation of the reduction basis for the flexible motion for each parameter iteration. Future work will focus on lowering the computational load of this stage through parametric model order reduction methods. Also the computation of full response derivatives through an adjoint approach are currently being developed.

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