A study of contact descriptions in the framework of the absolute nodal coordinate formulation

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Multibody system dynamics is a general approach which can be used to describe equations of motion in a straightforward manner and can be utilized for variety applications, such as granular dynamics, mechanical systems consisting of rigid and flexible components. Contacts between rigid and/or flexible bodies exist in various multibody applications such as belt drives, gears, bearings, human joints etc. The contact description within multibody dynamics still involves some challenge, especially in the case of thousands or millions of contacts in the dynamic system or in the case of very flexible bodies. The absolute nodal coordinate formulation (ANCF) is a finite element-based approach which is especially designed to be used as a part of multibody system dynamics approach. ANCF can predict the dynamic responses of flexible bodies subjected to large deformations in multibody applications [1].

The objective of this study is to analyze performance of different contact approaches in a framework of ANCF. In this work, the contact mechanics methods such as widely used penalty method [2] and a method for solving large cone complementary problems [3] are compared in case of simple two dimensional dynamic problems. Previously, the novel method of Cone Complementary Problems (CCP) has implemented for rigid body dynamics [3].

Clearly, small time steps which can achieve numerical stability is necessary when simulating multiple contacts. Therefore, an innovative time integration method to solve this class of problems is proposed by researchers, such as linear complementarity problem (LCP) and nonlinear complementarity problem (NCP) [3, 4, 5]. While when addressing a large number of contacts and polyhedral approximation used in friction [6], LCP and NCP solvers remain limitations. In this paper, a time integration scheme used in contact dynamics with CCP-method is studied.

The penalty approach is imposed through the Karush-Kuhn-Tucker condition which implies that when the bodies are not in contact, i.e. $g_N > 0$, then the pressure is zero, i.e. normal force $\lambda_N = 0$. In contrast, when the bodies are in contact, i.e. $g_N = 0$, then the pressure is induced between the bodies, i.e. normal force $\lambda_N < 0$. This formulation for normal contact is also known as Hertz-Signorini-Moreau condition.

The tangential part is constrained through the slip rate $(\dot{\gamma})$ and the slip condition (f_c). The slip condition, defined as

$$f_c = \|\boldsymbol{\lambda}_T\| - \mu|\lambda_N| \leq 0 \tag{1}$$

determines whether the bodies stick $(f_c < 0)$ or slip $(f_c = 0)$ on each other and $\boldsymbol{\lambda}_T$ is the tangential force. During the stick state, the net slip rate needs to be zero $(\dot{\gamma}) = 0$ and should be greater than zero during the slip state $(\dot{\gamma}) > 0$. The slip rate is related to the tangential gap through the direction of the tangential contact stress vector as

$$\dot{\boldsymbol{g}}_T = \dot{\gamma}\frac{\boldsymbol{\lambda}_T}{\|\boldsymbol{\lambda}_T\|} \tag{2}$$

The Cone Complementary Problem method represents the first order optimality conditions for the convex quadratic optimization problem with conic constraints as

$$\gamma^{(l+1)} = \min_{\gamma} \frac{1}{2}\gamma^T \boldsymbol{N}\gamma + \boldsymbol{p}^T\gamma \quad \forall\, \gamma_k \in \boldsymbol{C}_k \tag{3}$$

where $\boldsymbol{N}$ and $\boldsymbol{p}$ are constant symmetric positive matrix and constant vector which need to depend on the initial system of contact model and γ is the contact force at the contact point with the time step $t^{(l+1)} = t^{(l)} + \Delta t$. The Coulomb friction model $\boldsymbol{C}_k$ follows as:

$$i \in \mathscr{A}^{(l)}\left(\boldsymbol{e}(t), \Phi_i\right): \begin{cases} \gamma_{i,n}^{(l+1)} \geq 0 \\ \left(\mu_i \gamma_{i,n}^{(l+1)} - \gamma_{i,\tau}^{(l+1)}\right) \geq 0 \end{cases} \tag{4}$$

where $\boldsymbol{e}$ is the vector of nodal coordinates and $\gamma_{i,n}$ and $\gamma_{i,\tau}$ are components of contact force vector.

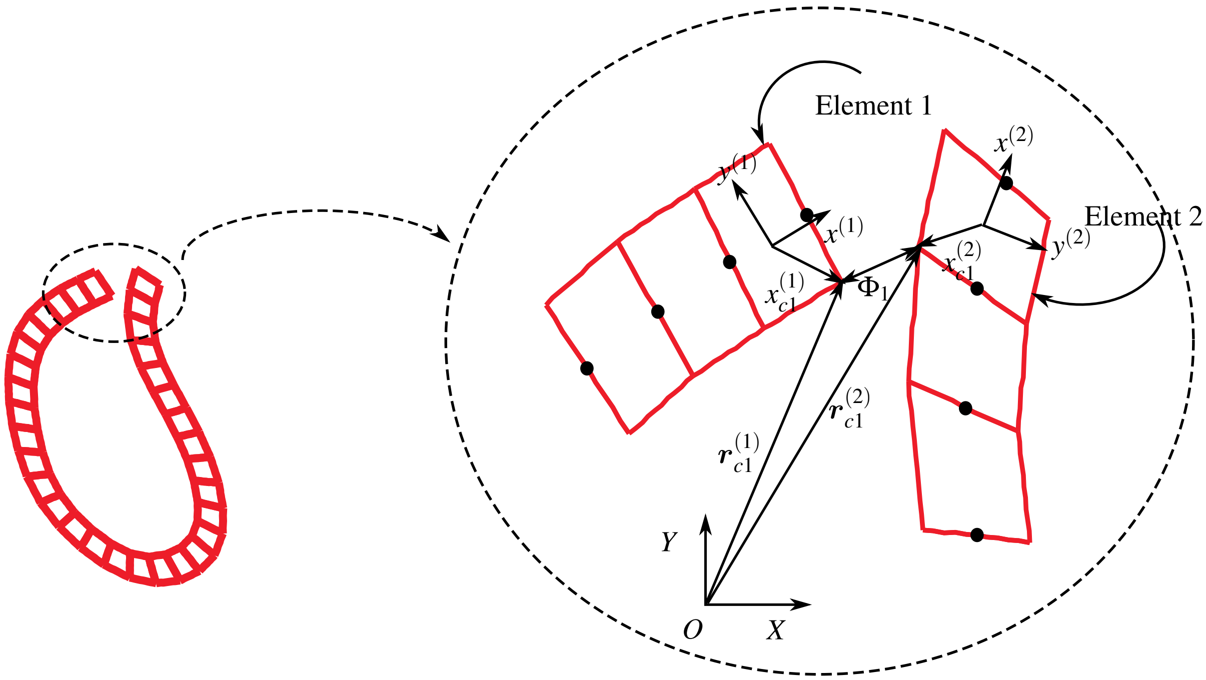

Fig. 1: Illustration of the elements 1 and 2 of the flexible beam in contact. An arbitrary contact point is located by the position vectors $\boldsymbol{r}^{(j)}$ via local constant vectors $\boldsymbol{x}_{c_1}$ and $\boldsymbol{x}_{c_1}$

As shown in Fig. 1 contact i can take a place between body A and B. In Fig. 1, Φ_i is the distance gap function between two contact points. When the two elements are in contact, they should not penetrate and thus a non-penetration constraint is imposed at these contact points as

$$\Phi_1 = \left\| \boldsymbol{r}_{c1}^{(1)} - \boldsymbol{r}_{c1}^{(2)} \right\| \tag{5}$$

where $\boldsymbol{r}^{(j)}$ is the position vector of the contact point and for each ANCF element of the beam, it can be given as $\boldsymbol{r}^{(j)} = \boldsymbol{S}_m \boldsymbol{e}^{(j)}$; $\boldsymbol{S}_m$ is the element shape function matrix; $\boldsymbol{e}^{(j)}$ is the vector of nodal coordinates of element j. However, it is worthy to note that finding of contact points for bodies with generic shapes is not trivial. For example, if the contact bodies are concave shape, there will be a large number of contact points, and it may be not possible to define the gap function.

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