Exact Workspace Synthesis of RCCR Spatial Mechanism for Task-Based Knee Rehabilitation

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This study is aimed at synthesizing a spatial mechanism for a task based knee rehabilitation exoskeleton device. An exact kinematic synthesis in the designing of linkages for an exoskeleton application is important in reducing the fitting and alignment issues. The desired trajectories of the knee joint have been obtained in this study through motion capture. The data is then used as an input to the synthesis procedure. Parallel mechanisms with a single degree-of-freedom (DOF) have been considered, and an RCCR mechanism with an exact workspace synthesis approach capable of generating the desired 3D motions of the lower leg is designed. The parameterized forward kinematics equations of each RC serial chain have been converted into implicit equations via elimination. The implicit description of the workspace is a function of the structural parameters of the serial chain, making it easy to relate those parameters to the motion capture data.

An option to relieve the symptoms and to restore the lost freedom for people with disability is rooted in assistive technologies. Despite significant advances in recent decades, existing assistive devices are constrained by limited portability, safety, ergonomics, autonomy, and cost. This is partly due to the attempts in aligning each robotic joint axis with its human counterpart (e.g., a hinge joint for the elbow and knee). This assumes a fixed human joint axis for the range of motion of the joint(s), which is not always the case [1]. The joint-to-joint alignment notion also influence the selection of mechanism types for upper and lower-body exoskeletons. Parallel robots have a great advantage in rigidity and in providing high precision. These classes of mechanism with systematic dimensional synthesis techniques can be utilized to realize robotic exoskeletons for the generation of human limb motions independent of the anatomical measures and landmark. Parallel robots workspace however, is limited by the existence of singularities. And most of the existing dimensional synthesis methods relay on finite positions of the trajectory, which may not ensure smoothness of motion between task positions. They may also be susceptible to circuit defects and singularities between task positions. Usually, to avoid such problems, additional optimization procedures are incorporated into the finite position-based synthesis [2]. In this study, while maintaining the simplicity of the finite positions approach, an exact workspace of the selected linkage [3], have been utilized to define the motion of the exoskeleton segments based on its point of attachment to the limb in order to insure smoothness of motion and avoid circuit defects and singularities.

The desired trajectory of the lower-leg has been recorded through motion capture, Fig. 1(a). The RCCR mechanism has been chosen due to its capability to create complex 3D motion with a single degree-of- freedom. In the synthesis, the parameterized forward kinematics equations of the RC serial chains have been formulated and converted into an implicit equation to eliminate the joint variables θ_1 and r_2. The implicit description of the workspace is made to be a function of the serial chains structural parameter and these parameters are related to the parameters of the desired trajectory. Fig. 1 (b) shows both the flexion/extension and pronation motions of the lower leg. In the synthesis however, the complex 3D pronation motion of the lower-leg is considered, as it provides the most rigorous exercise for the knee joint. Data points highlighted in green at Fig. 1(b) have been selected from the motion capture.

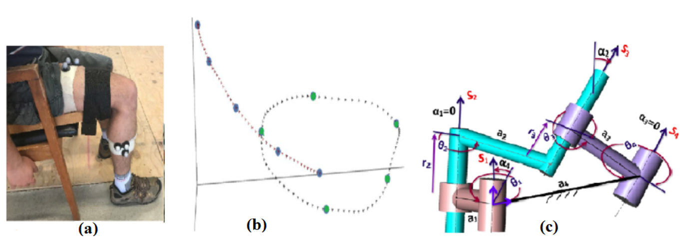

Fig. 1: (a) Markers placement for motion capture, (b) path and selected points, and (c) RCCR linkage and its structural parameters

According to the coordinate frame shown in Fig. 1(c), and applying the needed condition of parallel axes, that is, $\alpha_1 = \alpha_3 = 0$, the forward kinematics of both RC chains are,

$$[D_{RC1}] = \begin{bmatrix} c(\theta_1+\theta_2) & -s(\theta_1+\theta_2)c\alpha_2 & s(\theta_1+\theta_2)s\alpha_2 & a_2c(\theta_1+\theta_2)+a_1c\theta_1 \\ s(\theta_1+\theta_2) & c(\theta_1+\theta_2)c\alpha_2 & -c(\theta_1+\theta_2)s\alpha_2 & a_2s(\theta_1+\theta_2)+a_1s\theta_1 \\ 0 & s\alpha_2 & c\alpha_2 & r_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[D_{RC2}] = \begin{bmatrix} c(\theta_3+\theta_4) & s(\theta_3+\theta_4) & 0 & -a_3c\theta_4-a_4 \\ -s(\theta_3+\theta_4)c\alpha_4 & c(\theta_3+\theta_4)c\alpha_4 & s\alpha_4 & -r_3s\alpha_4+a_3s\theta_4c\alpha_4 \\ s(\theta_3+\theta_4)s\alpha_4 & -c(\theta_3+\theta_4)s\alpha_4 & c\alpha_4 & -r_3c\alpha_4-a_3s\theta_4s\alpha_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The elimination yields a quadratic surface expression,

$$Q(x,y,z) : (s_{1y}^2+s_{1z}^2)x^2+(s_{1x}^2+s_{1z}^2)y^2+(s_{1x}^2+s_{1y}^2)z^2-2s_{1x}s_{1y}xy-2s_{1x}s_{1z}xz-2s_{1y}s_{1z}yz+c_{21x}x+c_{21y}y+c_{21z}z=0$$

where (x,y,z) is a point of the $\mathbb{R}^3$ space of relative translations, $\mathbf{s}_1 = (s_{1x}, s_{1y}, s_{1z})$ is the direction for both joints, and $\mathbf{c}_{21} = \mathbf{c}_2 - \mathbf{c}_1 = (c_{21x}, c_{21y}, c_{21z})$ is the vector along the common normal between both joints. This surface is classified as a circular cylinder, with radius $R = \sqrt{\mathbf{c}_{21} \cdot \mathbf{c}_{21}}$. The points obtained from the motion capture for the lower-leg motion have been utilized to solve this quadratic equation ($Q(P_i) = 0,\ i = 2,3,4,5$). The intersection of two such circular cylinders yields a quartic curve which is the workspace of the RCCR linkage.

Following the outlined exact synthesis procedure, different sets of solutions have been obtained. Any two combination of those solutions can be combined to generate the workspace of the RCCR linkage. These solutions can easily be examined for circuit defect and smoothness of the trajectory. For example, two solutions are shown in Fig. 2(a) &(b), out of which, Fig.2(a) has a circuit defect, while it has the exact locations of the desired points from the motion capture. Figure 2(b) shows the solution with no circuit defect and Fig. 2(c) blue, shows its comparison with the desired trajectory obtained through the motion capture, Fig. 2(c) red. The two trajectories have shown similar profile, considering the original trajectory from the motion capture is a representative of several paths with few offsets. The mechanism has been modeled and simulated in a CAD environment as shown in Fig. 2 (d).

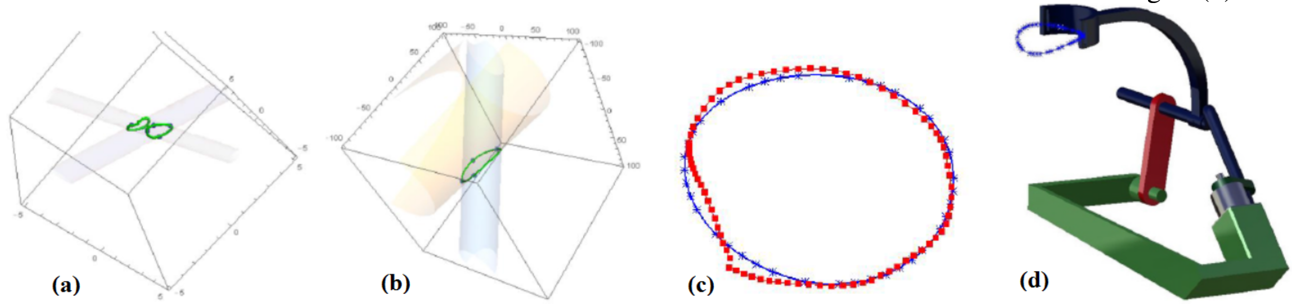

Fig. 2: (a) solution with circuit defect, (b) solution with no circuit defect (c) trajectory (red-from motion capture, blue-from mechanism), and (d) CAD model of the selected solution

Overall, in this study, exact kinematics synthesis has been utilized to synthesize a 1- dof RCCR spatial mechanism for task-based knee rehabilitation. As the synthesis utilized the end-effector attachment trajectory, there was no need for the knee joint to have an alignment with its counter part mechanism joint. In other words, the joints of the mechanism have been located independent of the anatomical landmarks of the knee joint. The entire workspace of the RCCR mechanism has been considered in the selection of the synthesis results, to insure smoothness of motion between task positions and to generate the desired complex 3D trajectory of the lower-leg. The mechanism has also been further analyzed and simulated with its dynamic model and control interaction with the user.

References

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