An Efficient High-precision Recursive Algorithm for Net Multibody Systems

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With the development of multibody system dynamics, the research on chain systems, tree systems and simple closed-loop systems has been very mature so far. Lots of relevant algorithms have been proposed, from traditional algorithms such as the Newton-Euler method, the Lagrangian equation, the Kane method and so on, to the O(n) complexity recursive algorithms like the articulated-body algorithm (ABA), the spatial operator algebra (SOA) and the divide-and-conquer algorithm (DCA). When the parallel processors are sufficient, the DCA can even achieve O(log(n)) complexity. However, the studies about net multibody systems and other complex topological systems are extremely few.

In the net multibody system, bodies are always connected by high pairs, and the number of closed-loop constrains is O(n) or even more. Thus, the dynamic equations are complicated differential / algebraic equations (DAEs), and cannot be solved by usual ODE integrators which will cause location or speed constraint violation problems. This also means the present O(n) complexity algorithms such as ABA, SOA and DCA will not be appropriate for the net multibody system. We can only utilize the DAE integrators, for instance, the Newmark integrator, the HHT-I3 integrator, the generalized \(\alpha\) integrator, but this needs to solve the Jacobian matrixes of the system, which makes the computational process become more difficult and extremely slow. Therefore, how to accurately and efficiently solve dynamic problems of net multibody systems is an urgent task. In this paper, we propose a new algorithm to simulate net multibody systems, with O(n) complexity and without any constraint violation problems.

Considering bodies are always connected by high pairs in the net system, using traditional position description methods, for example, the Lagrangian coordinate and the Cartesian coordinate, will increase lots of constraints. Thus, a new position description method called “the hybrid nodal coordinate method” is proposed in this paper, which uses the absolute coordinates of two endpoints, a spin angle and a length constraint to describe the rigid body motion, as shown in Fig.1. The hybrid nodal coordinate method can translate rigid body motion to node motion, and hugely decrease the number of constraints when used in the system connected by high pairs. Another advantage of this method is the dynamic equation is extremely simple with a constant generalized mass matrix.

Define the row and line direction of the net system and number bodies sequentially, we can deduce the dynamic equation (DAE form) based on the Newton-Euler method. Then, we use the Newmark integrator to solve
the DAE, and a difference equation will be generated. According to the topological structure of the difference mass matrix, we can recursively solve the difference equation by appropriate matrix manipulation. The whole method has an O(n) computation complexity and does not have any constraint violation problems.

As shown in Fig.2, a net multibody system is constituted of 60 rods connected by all spherical joints. The system is initially in the horizontal position and swings back and forth under gravity. We set the simulation step size as 0.001s and the simulation time as 2s. The resulting configurations of the system at different moments are shown in the Fig.3, which is in good agreement with the MSC.Adams. As shown in Fig.4, the joint clearance curve proves that this method has a good control over the position constraint. We also use the DOF-CPU time curve to verify this method is an O(n) complexity algorithm.

![Fig. 3: The configurations of the system at different moments](image1)

![Fig. 4: The location constraint error of the joint (blue circle in the Fig.2)](image2)