Geometric Modeling of Flapping Wing Dynamics in Lie Group Setting

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In order to study dynamics of flapping wing moving at low Reynolds number in ambient fluid, we adopt geometric modelling approach of fully coupled wing-fluid system, incorporating Boundary Integral Method for calculating added masses, and Lie group rigid body integrator. Our aim is to explore numerical advantages of such an approach in comparison to the standard procedures that comprise volume discretization of a fluid domain. The configuration space of a flapping wing is modelled as a Lie group

$$G = \mathbb{R}^2 \times SO(2) \tag{1}$$

with the elements of the form $p = (x, \mathbf{R})$. The left multiplication in the group is given as $L_p\!:G \rightarrow G,\ p \rightarrow p \cdot p$, where the identity element of G is defined by $e = (\mathbf{0}, \mathbf{I})$ where $\mathbf{0}$ and $\mathbf{I}$ are null and identity matrices, respectivelly. With G so defined, its Lie algebra with the element v comprising wing velocities ('linear' and angular) is given as [1]

$$g = \mathbb{R}^2 \times so(2). \tag{2}$$

By assuming potential flow of an ideal fluid - i.e. inviscid, incompressible fluid with irrotational flow - the configuration space of the coupled wing-fluid system is reduced by eliminating fluid variables via symplectic and Lie-Poisson two stages reduction [2, 3], identifying configuration space with G and reducing dynamics to g^* (dual to g). The first reduction exploits particle relabeling symmetry, associated with the conservation of circulation for incompressible ideal fluid (fluid kinetic energy, fluid Lagrangian and associated momentum map are invariant with respect to this symmetry [4]). Consequently, the equations of motion for the whole system can be formulated without explicitly incorporating the fluid variables [5]. The effect of the fluid flow to the wing overall dynamics is accounted for by the added masses [6], and - as a result - the system is parameterized via p and v only. In such approach, the added masses are expressed as boundary integral functions [7] of the fluid density and velocity potentials that are, in turn, functions of p and v. After particle relabeling symmetry, further reduction is associated with the invariance of the dynamics under superimposed rigid motions.

The two test cases will be considered here. First test case is an example of the wing with a blunt edge, submerged in an ideal fluid, which is at rest at the infinity. At any time t, rigid wing and fluid occupy an open connected region $\mathcal{M}$ of the Euclidean space, identified here with $\mathbb{R}^2$. More specifically, the body occupies region $\mathcal{B}$ and the fluid occupies a connected region $\mathcal{F} \subset \mathcal{M}$ such that $\mathcal{M}$ can be written as a disjoint union of open sets as $\mathcal{M} = \mathcal{B} \cup \mathcal{F}$. As the region $\mathcal{F}$ is connected and flow is irrotational, the velocity field v can be expressed in terms of a potential $v = \nabla\phi$ and incompressibility implies that the Laplacian of ϕ is zero [4], i.e. $\Delta\phi = 0$ in $\mathcal{F}$. Furthermore, with a prescribed velocity on the boundaries, Neumann problem for the Laplace equation can be formulated and boundary conditions numerically imposed [8]

$$\begin{gathered} \nabla\phi \cdot n = v \cdot n \ \text{ on } \ \partial\mathcal{B}, \\ \Delta\phi = 0 \ \text{ at } \ \infty. \end{gathered} \tag{3}$$

Another test case that will be presented is a wing-fluid system that includes rigid wing with sharp edge. Here, in order to account for important effects of vorticity in the fluid flow around the wing, the vortices will be shed from the sharp edge by enforcing a Kutta condition in every time step [9, 10].

With the aim of dynamics integration of a rigid wing immersed in ambient fluid, Lie group integrator that operates in state space [1] will be used. To this end, rigid wing state space is introduced as

$$S = G \times g \quad \text{i.e. } S = \mathbb{R}^2 \times SO(2) \times \mathbb{R}^2 \times so(2) \cong TG. \tag{4}$$

This is a Lie group itself that possesses Lie algebra

$$s = \mathbb{R}^2 \times so(2) \times \mathbb{R}^2 \times \mathbb{R}^2. \tag{5}$$

As it is shown in [1], DAE-index-1 Lie group integrator can be utilized for dynamics integration in such a setting. This type of integrator, but supplemented with the added masses determination procedure based on the properties of fluid flow, will be used for solving dynamics of the flapping wing test cases.

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