Nonlinear state estimation in flexible-link multibody systems through reduced-order models

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In the last years, the use of flexible link multibody systems (FLMBs) is boosted by a growing sensibility to sustainability and energy saving. The accuracy and the repeatability of these systems can be compromised by their flexibility that often causes unwanted vibration. Therefore, the synthesis of effective control scheme is necessary for vibration suppression. Most of these controllers are typically model-based and require the knowledge of all the system state variables (i.e. position and velocity of each coordinate), or at least to a meaningful set of them, to ensure effective control of the infinite number of degrees of freedom (dofs) that characterize such a kind of system. Since the direct measurement of all the state variables is almost impossible, state observers should be employed to estimate the unmeasured state variables that are of interest for control. These state variables are usually a slightly smaller subset of the whole state vector employed at the modeling stage, and are those having prominent observability and controllability \cite{1}. Therefore, state observers based on effective reduced order dynamic models should be employed to ensure reliable estimation of the most important state variables while reducing the computational effort.

This paper shows some preliminary results on state estimation in FLMBs through nonlinear reduced order dynamic models formulated through independent coordinates (and hence through a set of ordinary differential equations). The mechanism shown in Fig. 1 is assumed as the test case. The observer is synthesized through the EKF algorithm and the reduced order model presented in \cite{1}, where a 30-dof model has been obtained through the Equivalent Rigid Link System approach. Such a full order model has been then reduced through a modified Craig-Bampton strategy leading to a nonlinear 13 dofs model, containing 9 physical coordinates (i.e. the master dofs shown in red in Fig.1) and 4 interior modal coordinates. The resulting state vector of the first-order model used for estimation has therefore 26 variables.

![Fig. 1: Studied flexible link manipulator and variables involved in the estimation process](image-url)
Three measured torques exerted by the three motors driving the system (T₁, T₂, and T₃ in Fig. 1) have been used as the measured inputs of the state observers. Additionally, six sensed outputs have been employed as the model output adopted to compute the observer innovation: i.e. the angular positions of the three-actuated links (q₁, q₂, and q₃) and the curvatures (strains) of the midpoints of links 1, 2 and 4 (see Fig. 1). The observability analysis has been computed employing a linearized observability analysis in the whole manipulator workspace and it proves that the selected set of input and output variables guarantees system observability.

The validation of the observer is carried out by means of a simulated test in Matlab. The motion of the reference manipulator, which can be thought of as the “real mechanism”, is simulated through the full-order model. In contrast, the observer is based on the nonlinear reduced-order manipulator model. Additionally, the signals of the measured input and output fed to the observer are corrupted with noise. Such a test allows for an effective assessment of the observer outcomes by comparing all the manipulator state variables, including the elastic ones which cannot be measured experimentally, and by properly evaluating the impact of the model truncation in the estimation. A simulation lasting 3 seconds has been tested.

As an example of estimates, Fig. 2 shows the estimated elastic displacements and velocities of the manipulator tip. The same figure also shows the time-histories of the error between the actual variables (computed through the full-order model and free measurement error signals) and the estimated ones. Error diagrams clearly show that the observer is stable and is able to deliver accurate estimates of the elastic state variables, as well as of the manipulator gross motion, not show here for sake of space. These preliminary results prove that both the reduction strategy and the estimation scheme are effective.

![Fig. 2: Estimated linear displacements (a,c) and velocities (b,d) of node 12 in x direction (a,b) and y direction (c,d). Position (e,g) and velocity (f,h) estimation errors.](image)

References
