Sensitivity Analysis of Multibody System Dynamics Based on L-Stable Method

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Sensitivity analysis provides gradient information that is required by local optimization strategies, and is often discussed in the context of design optimization. Since second order sensitivity can improve convergence efficiency of optimization algorithms and can provide more accurate information, Haug et al[1] study the second order design sensitivity analysis of multibody systems described by first order ordinary differential equations. Ding et al[2] develop the second order sensitivity analysis of multibody systems described by differential-algebraic equations and present a hybrid method combines the direct differentiation method and the adjoint variable method.

Direct differentiation method and adjoint variable method are major analytical techniques for sensitivity analysis applied to multibody optimal design, and the hybrid method is a combination of the two major methods. The direct differentiation method can be easily implemented by straightforward differentiation of system governing equations, performance measures, and/or constraint equations with respect to design variables, while the adjoint variable method is to calculate the design sensitivity after solving a set of adjoint variable equations derived from variations of system equations, which can avoid direct calculations of state derivatives with respect to design parameters.

During the process of sensitivity analysis of multibody systems, there is an important part that is the solution of the motion equations. The equations of motion are generally formulated as index-3 differential-algebraic equations (DAEs). Efficient and stable numerical method for DAEs became one of the key problems in multibody dynamics. In nowadays, many numerical methods are discussed for the DAEs, such as HHT method, generalized-α method etc. Wu[3] discussed a L-stability block method based on Taylor expansion for structural dynamics equations and multibody dynamics equations. In this paper, the L-stable method is developed for index-3 DAEs, and is used for the hybrid sensitivity analysis.

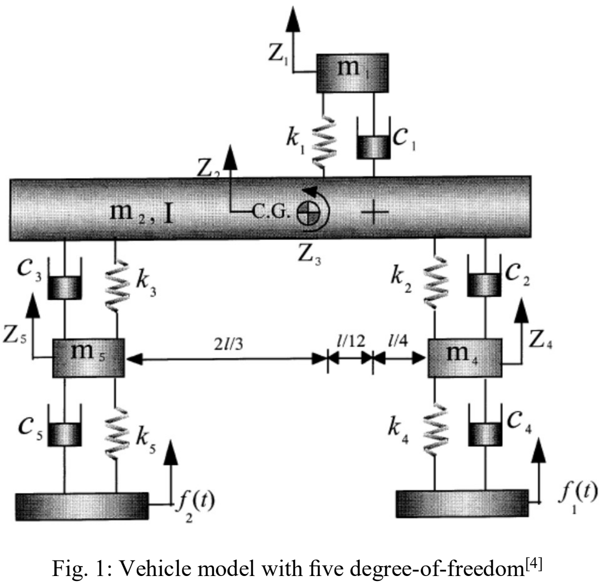

Fig. 1: Vehicle model with five degree-of-freedom[4]

Figure 1 is a vehicle model with five degree-of-freedom(DOF), the displacement functions of the front and rear wheels are

$$f_1(t) = \begin{cases} \overline{v}(t) & 0 \le t \le T \\ 0 & others \end{cases}, \ f_2(t) = f_1(t-\tau), \ \tau \le t \le T+\tau \tag{1}$$

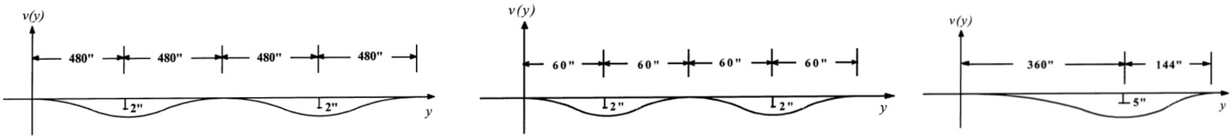

Fig. 2: Different road conditions [4]

By using the L-stable method presented, the first and second sensitivity analysis of the design parameters $\boldsymbol{b} = [k_1 \ k_2 \ k_3 \ c_1 \ c_2 \ c_3]^{\mathrm{T}}$ with respect to the objective functions on different road conditions are obtained. The results are compared with those obtained from other numerical methods such as Runge-Kutta method and generalized-α method.

Acknowledgments

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