The Shimmy Phenomenon in Dynamics of Driven Rigid Castor Wheel

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We study the dynamics of a Castor wheel driven by a constant force. The wheel touches the horizontal plane: depending on an interaction model different dynamical effects appear. We explore three models: ideal non-holonomic constraint, dissipative non-holonomic constraint and dry friction in the contact patch between the wheel and the plane. For both models of non-holonomic constraint, we plot the phase portrait of the system and find the domains where the constraint reaction belongs to the friction cone. The third model is a model of the deformable rough plane described in [1]. Due to differential form of Coulomb dry friction and dynamic distribution of normal stresses in the contact patch, we obtain the model that produces friction forces and torques depending both on slipping velocity and angular velocity of the wheel. Numerical simulations show that for some combinations of the parameters the rectilinear motion becomes unstable and the oscillations appear. This effect is known as shimmy phenomenon [2].

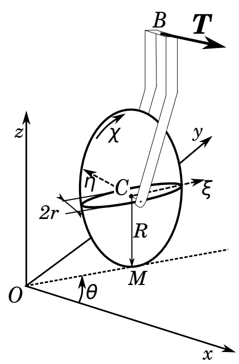

Fig. 1: Statement of the problem

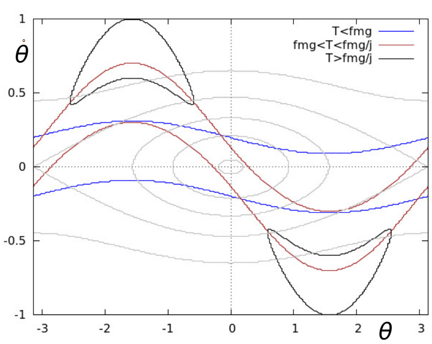

Fig. 2: Phase portrait for ideal non-holonomic constraint with slipping areas.

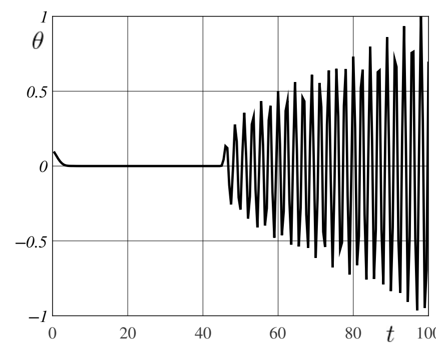

Fig. 3: Shimmy for the motion.

Absolutely rigid wheel of mass m and radius R rotates about horizontal axis $C\eta$ with respect to the massless fork. The force $\mathbf{T}$ that is constant in the inertial coordinate frame is applied to the fork in the point B (Fig. 1). The offset — the distance between the projections of the points C and B – is denoted by b. The coordinates of the system are: the coordinates $x,\ y,\ z$ of the wheel's center C on the supporting plane, the course angle θ between the inertial axis Ox and the central plane of the wheel and the angle of the wheel's proper rotation χ. The axes $C\xi\eta z$ are attached to the fork, and $v_\xi,\ v_\eta, v_\zeta$ are the components of the point C's velocity in this frame.

The dynamic equations of motion for arbitrary model of the wheel-plane interaction are

$$\begin{aligned} \dot z &= v_z, & m\dot v_z &= F_z - mg \\ m(\dot v_\xi - v_\eta \dot\theta) &= F_\xi + T\cos\theta, & m(\dot v_\eta + v_\xi\dot\theta) &= F_\eta - T\sin\theta \\ J_1\ddot\theta &= M_z - bT\sin\theta, & J_3\ddot\chi &= -F_\xi R + M_\eta \end{aligned}$$

Here g is gravity acceleration, J_1 and J_3 are equatorial and axial momets of inertia, F_ξ, F_η, F_z are the projections of the friction force and normal reaction, M_η, M_z are rolling and spinning friction torques with respect to the point M.

If the wheel does not slip and the supporting plane is absolutely rigid then we have

$$v_\xi - R\dot\chi = \dot x\cos\theta + \dot y\sin\theta - R\dot\chi = 0,\ v_\eta = \dot x\sin\theta - \dot y\cos\theta = 0,\ v_z = \dot z = 0$$

for velocities and coordinates respectively. These contraints allow calculating of reactions and reducing the order of the system. Thus, for ideal constraints the dynamics is governed by two separable ODE of second order, and the first one is the mathematical pendulum equation:

$$\begin{aligned} J_1\ddot{\theta} + bT\sin\theta &= 0 \\ (J_3+mR^2)\ddot{\chi} &= TR\cos\theta \end{aligned} \tag{1}$$

The components of the reaction are

$$F_z = mg,\ F_\xi = -jT\cos\theta,\ F_\eta = T\sin\theta + mR\dot{\chi}\dot{\theta}, \quad j = J_3/(J_3+mR^2) \leq \frac{1}{2}$$

The wheel does not slip if the reaction belongs to the friction cone:

$$\sqrt{F_\xi^2 + F_\eta^2} \leq fF_z$$

The phase portrait of ODE (1) and the areas of slipping for some fixed coefficient f of dry friction and fixed velocity $\dot{\chi}$ is presented on Fig. 2. We obtain that if $0 < T < fmg$, then for a fixed value $\dot{\chi}$ the wheel does not slip into the region bounded by curves that do not intersect the axis $\dot{\theta} = 0$. This region contains a stable stationary motion and small oscillations in its vicinity. If $fmg \leq T \leq j^{-1}fmg$, then the region of motion without slipping is a centrally symmetric figure. When $T > j^{-1}fmg$, then a motion without slippage is possible inside a closed region in the neighborhood $\theta = \pm\pi/2, \dot{\theta} = \mp 0.7$ and do not contain full trajectories. However, with the growth of $\dot{\chi}$ these areas are tighten to the axis $\dot{\theta} = 0$. Therefore, for any arbitrarily small oscillation $\theta(t)$ in a neighborhood of a stationary motion $\theta = 0$, $\chi = \frac{TR}{J_3+mR^2}\frac{t^2}{2}$ after some time the wheel begins to slip.

However, if we include dissipative spinnig and rolling torque of viscous type

$$M_\eta = -c_1\dot{\chi},\ M_z = -c_2\dot{\theta}$$

then the equations of motion are

$$\begin{aligned} J_1\ddot{\theta} + bT\sin\theta &= -c_2\dot{\theta} \\ (J_3+mR^2)\ddot{\chi} &= TR\cos\theta - c_1\dot{\chi} \end{aligned}$$

and the solution $\theta = 0$ becomes assimptotically stable. The velocity of proper wheel rotation is bounded:

$$\dot{\chi} = \frac{TR}{c_1} + \left(\dot{\chi}_0 - \frac{TR}{c_1}\right)\exp\left(-\frac{c_1 j}{J_3}t\right) \leq \max\left(\frac{TR}{c_1}, \dot{\chi}_0\right)$$

It means that for some value of driven force T there exist the vicinity of the point $\theta = 0$ such that the motion starting there asymptotically tends to rectilinear motion and slippage do not occur.

The results obtained for non-holonomic constraint is interesting to compare with that for the system with sliding. We take the model of distributed dry friction [1]: it gives all types of friction (sliding, rolling, spinning) that depends both on $\mathbf{v}_M$ and $\boldsymbol{\omega} = \dot{\theta}\mathbf{e}_z + \dot{\chi}\mathbf{e}_\eta$. Numerical simulations show that for some values of parameters, the rectilinear motion of the wheel is stable for small values of $\dot{\chi}$ and unstable for large $\dot{\chi}$. On Fig. 3, the wheel of mass $m = 5$ starts with zero speed $\dot{\chi}$ and $\theta(0) = 0.1$ rad. The driven force $T = 5$ and the coefficient of differential Coulomb law in the contact patch is $f = 0.2$. While the speed is small, θ fastly decreases to zero, but after approximately 45 time-units the oscillation begins and grows signicantly up to a certain finite amplitude.

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References

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