

# Elimination Method for Parallelization of Flexible Multibody System Dynamics with Kinematical Loops

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The most effective methods for the parallel multibody dynamics solution are the ones with the logarithmic complexity. This paper describes the extension of the advanced procedure [1-2] of the parallelized dynamic solution of flexible multibody system from the serial kinematic chain into the multibody systems with kinematical loops.

**Previous work.** The basic method is based on the modified state space and the efficient set of natural coordinates [1-2] and modal coordinates for the description of the deformation in the system. The equation of motion for single body  $i$  is derived as

$$M_i \ddot{s}_i + K_i s_i = Q_i \tag{1}$$

where  $M_i$  is the mass matrix,  $K_i$  is the stiffness matrix and  $Q_i$  is the vector of generalized forces. Using the procedure described in [1-2] exploiting the Schur complement the resulting system of equations of motion (EOM) is obtained

$$M \dot{s} + J^T \mu = p^* \tag{2}$$

$$J \dot{s} = -\alpha f(s) \tag{3}$$

where  $M$  is the diagonal mass matrix,  $J$  is the Jacobi matrix corresponding to the constraints  $f$ ,  $\alpha$  is the coefficient of the Baumgarte stabilization,  $s$  is the vector of natural coordinates describing the absolute system position,  $p^*$  is the modified momentum of the system and  $\mu$  is the vector of the new Lagrange multipliers. Expressing  $\dot{s}$  from (2) and substituting into (3) the resulting system for unknown  $\mu$  is obtained

$$JM^{-1}J^T \mu = \alpha f(s) + JM^{-1}p^* \tag{4}$$

which can be simply written as follows

$$A \mu = b \tag{5}$$

The system of equations (5) is sparse, symmetric, positive definite with band structure for the case of a simple kinematic chain of  $n$  bodies (Fig. 1 a). The system (4) has a structure of blocks (Fig. 1 b) corresponding to particular bodies with equivalent (small) sizes.

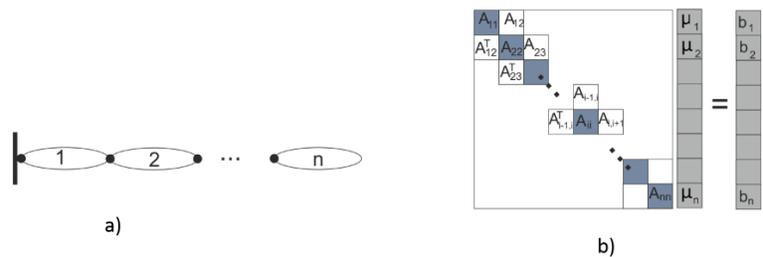


Figure 1 a): The simple kinematic chain    b) The resulting matrix-structure

Thus the whole system of the equations can be understood as a set of the interconnected subsystems representing by the blocks for unknown vectors  $\mu_i$ . The number of elimination levels is  $\log_2(n)$  and this is proportional to the resulting computational costs of elimination process. Based on the comparison of the application of elimination process and Cholesky decomposition the combination of both approaches has been proposed [1-2]. The result is the efficient combination of elimination process and Cholmod procedure (Fig.2). This

combination has been investigated for the small blocks (9x9). In the case, that the 9x9 division is used and there are not enough processors for the matrix transformations, the process is following. The number of subsystems  $n_s$  for elimination is the same as the number of bodies  $n$ . The number of processors  $n_p$  is smaller than  $n$ . Therefore it is possible to evaluate only  $n_p$  elimination in parallel on one elimination level and the rest has to be carried out after that.

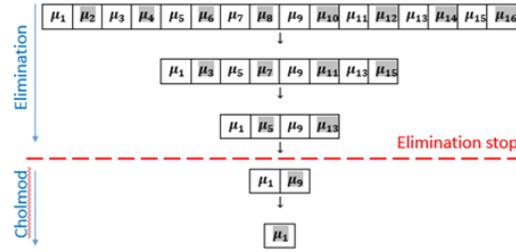


Figure 2: Efficient combination of the elimination with the Cholesky decomposition

It is obvious, that the system of equations (5) can be split into the sub-blocks which number corresponds to the number of processors ( $n_s = n_p$ ). However, it is always better from the complexity point of view to split the system in that way, that the number of sub-blocks corresponds to the number of particular bodies in the kinematical system ( $n_s = n$ ). Thus the optimal elimination process is obtained, see [2]. The complexity of the solution is very promising. The example in [2] demonstrates that 10 times increased of efficiency is possible even on 10 processors for small flexible multibody systems (20 bodies with 10 flexible modes).

**New contribution.** The limitation of this approach is that it is valid just for kinematic chain or branched kinematic tree. The problem is the occurrence of kinematic loop that leads to matrix structure from Fig. 1b with broader bandwidth. This paper is devoted to the description of solution of this problem.

The approach for solving the multibody systems with kinematic loops consist in the dividing suitable body within the loop into two bodies and their firm connection. The connection is described by appropriate constraint within the constraints  $f(s)$  in (3). The only requirement is that the distance of coordinates describing the position of mutually constrained bodies in the enumeration of coordinates in the vector  $s$  is small. This corresponds to the resulting bandwidth of matrix in Fig. 1b. This can be achieved by repeated division of bodies. This leads to the increase of bodies and coordinates and potentially the computational costs. However, the computational costs depends on the number of bodies logarithmically and thus the increase of number of bodies is negligible. If the number of bodies is for example increased 2 times then the computational costs are increased just by  $\log_2(2n) = 1 + \log_2(n)$ .

The example is in Fig. 3. The simple kinematical loop is in Fig. 3a. Its representation in the order of coordinates is in Fig. 3b. The reduction of distance of constrained coordinates is depicted in Fig. 3c. It is achieved by the dividing the first body in the order into three ones and introducing equivalent constraints. The resulting distance of constrained coordinates is reduced from 6 to 3.

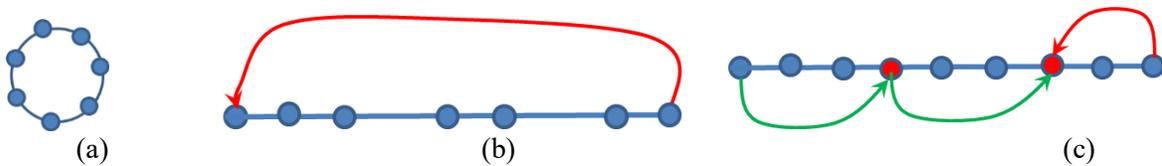


Figure 3: (a) The simple kinematical loop (b) The order of coordinates (c) Solution with added bodies

The resulting computational complexity is very promising as the very advantageous computational complexity for multibody systems with just kinematic chains is extended towards multibody systems with kinematic loops.

## References

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