A Consistent Treatment of Boundary Conditions for Fluid-Solid Interaction Problems

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Introduction. Smoothed Particle Hydrodynamics (SPH), which is used to discretize the Navier-Stokes equations, is a Lagrangian method frequently used for handling Fluid-Solid Interaction (FSI) problems [1, 2]. While having certain attractive features, the handling of boundary conditions in SPH is more challenging than in Eulerian approaches such as the Finite Volume (FV). Given that the accuracy of enforcing boundary conditions at the interface of fluid and solid phases directly dictates the accuracy of the FSI solution, in this abstract we demonstrate that the enforcement of boundary conditions (BC) frequently used in SPH underperforms the techniques used in FV. In the full-paper submission we will illustrate how integrating ideas from the FV solution into the SPH framework improves the quality of the latter.

Method. The governing equations of interest are the incompressible Navier-Stokes (NS),

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla P + \nu \nabla^2 \mathbf{u} \,, \tag{1}$$

where $\mathbf{u}$, ρ, P and ν are the velocity, density, pressure and kinematic viscosity of the fluid. The NS equation, along with the mass balance equation, govern the motion of fluids, which is a critical part in studying FSI problems. The NS equations are discretized in space and integrated in time in order to characterize the evolution of the system. A projection approach, introduced by Chorin [3] and shown in Eq.1, is a widely used time integration scheme for solving incompressible flow equations. The method decouples pressure from velocity as follows:

$$\begin{aligned} \frac{\mathbf{u}^* - \mathbf{u}^n}{\Delta t} &= -\mathbf{u}^n \cdot \nabla \mathbf{u}^n + \frac{\nu}{2}(\nabla^2 \mathbf{u}^n + \nabla^2 \mathbf{u}^*), & (2) \\ \nabla^2 P^{n+1} &= \frac{\rho}{\Delta t} \nabla \cdot \mathbf{u}^*, & (3) \\ \frac{\mathbf{u}^{n+1} - \mathbf{u}^*}{\Delta t} &= -\frac{1}{\rho} \nabla P^{n+1}, & (4) \end{aligned}$$

where $\mathbf{u}^*$ denotes the prediction velocity. Solving Eq.2 requires knowledge of boundary conditions for $\mathbf{u}^*$. In earlier Computational Fluid Dynamics (CFD) methods [3], the velocity boundary condition at $\mathbf{u}^n$ is used as the boundary condition for $\mathbf{u}^*$. This choice is not consistent with $\mathbf{u}^*$, in that $\mathbf{u}^*$ should not include the contribution of the pressure gradient term. When employed, this approximation limits the accuracy of the time integration to $O(\Delta t)$ [4]. Significant research has been conducted to address this aspect [5]. Inspired by this work, we use a high-order accurate boundary condition for $\mathbf{u}^*$ equation, Eq.2, that leads to a consistent formulation on the boundary. We add the contribution of the pressure gradient at the boundary, making it consistent with Eq.4 on the domain:

$$\mathbf{u}^*_{\partial D} = \mathbf{u}^{n+1}_{\partial D} + \frac{\Delta t}{\rho} \nabla P_{\partial D}. \tag{5}$$

In many SPH formulations, boundary conditions are imposed using ghost Boundary Condition Enforcing (BCE) markers [6, 2]. Then, the velocity of BCE markers a, $\mathbf{v}_a$, is obtained by enforcing no-slip condition [6]:

$$\mathbf{v}_a = 2\mathbf{v}^p_a - \tilde{\mathbf{v}}_a, \tag{6}$$

where $\mathbf{v}^p_a$ is the prescribed wall velocity at the position of SPH marker a, and $\tilde{\mathbf{v}}_a$ is an extrapolation of the smoothed velocity field of the fluid phase to the BCE markers, $\tilde{\mathbf{v}}_a = \sum_{b \in \mathbf{F}_a} \mathbf{v}_b W_{ab} / \sum_{b \in \mathbf{F}_a} W_{ab}$. Here, $\mathbf{F}_a$ denotes a set of "fluid" markers that are within the compact support of the BCE marker a and W_{ab} is the kernel, or weight, function. As seen in Eq.6, the velocity boundary condition does not include the pressure gradient term used in Eq.5. Hence, it stands to reason that this boundary conditions formulation will under-perform the consistent formulation of Eq.5.

Results. Our ultimate goal is to improve the SPH-based FSI implementation in Chrono [**?**]. To that end, we borrow from the FV solution the choice of time discretization and BC enforcement. The new SPH results are shown in Fig. 1 for a backward facing step problem. For reference, we compare the FV results obtained via Eq.5, see Fig. 2, against the old SPH solution that draws on Eq.6, see Fig. 3. We note that the SPH integration time step when using the consistent formulation is about one order-of-magnitude larger. For the full paper, we plan to demonstrate that implementing the proposed boundary conditions of Eq.5 yields a more robust and efficient SPH-based FSI solver.

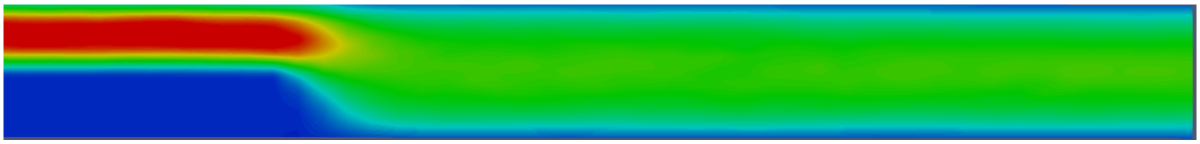

Fig. 1: Backward facing step results of the SPH projection method with the consistent formulation.

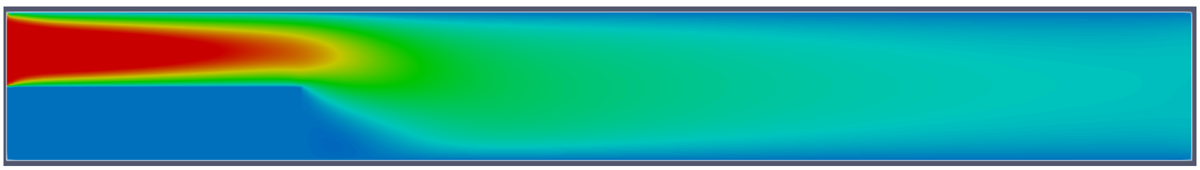

Fig. 2: Backward facing step results of the projection method with consistent formulation using FV.

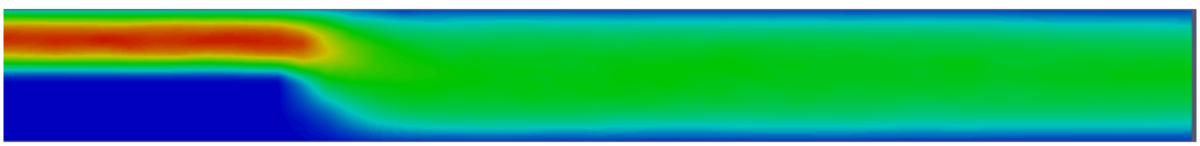

Fig. 3: Backward facing step results for the SPH projection method without the consistent formulation of Eq.5.

Conclusion. When presented a consistent boundary condition formulation for the projection method that augments the temporal and spatial accuracy. This formulation is consistent with the two-step projection method and therefore making use of this form will increase the performance of an SPH solver. In terms of computational performance, we found that the finite volume based solver executed 2.5 times faster than the SPH code. Ultimately, this treatment will improve the robustness of FSI solvers such as the one presented in [2].

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