Formulations of Viscoelastic Constitutive Laws for Beams in Flexible Multibody Dynamics

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A beam is defined as a structure having one of its dimensions much larger than the other two. The generally curved axis of the beam is defined along that longer dimension and the cross-section slides along this axis. The cross-section's geometric and physical properties are assumed to vary smoothly along the beam's span. Numerous components found in flexible multibody systems are beam-like structures: linkages, transmission shafts, robotic arms, etc.

The analysis of complex cross-sections featuring composite materials and the determination of the associated sectional properties was first presented by Giavotto *et al.* [1]. Their approach, based on linear elasticity theory, leads to a two-dimensional analysis of the beam's cross-section using finite elements and yields the sectional stiffness characteristics in the form of a 6×6 stiffness matrix relating the six sectional deformations, three strains and three curvatures, to the sectional loads, three forces and three moments. Furthermore, the three-dimensional strain field at all points of the cross-section can be recovered once the sectional strains are known.

The development of inelastic constitutive material laws for nonlinear beams has received far less attention than that of elastic laws. In numerous publications, damping terms have been added in an ad-hoc manner to *linear* Euler–Bernoulli or Timoshenko beam models (usually assumed to have a straight reference geometry); in contrast, viscous damping models for *geometrically nonlinear* beams or rods are discussed only rarely. The few articles that have appeared in the computational mechanics literature are discussed briefly.

Simo *et al.* [2] formulated sectional-level viscoplastic constitutive laws for geometrically exact rods without resorting to local, three-dimensional constitutive laws. In contrast, Mata *et al.* [3] used the kinematic model of Simo [4] to develop inelastic constitutive behaviour laws for composite beam structures under dynamic loading. They evaluated the inelastic stresses by numerical integration of three-dimensional Piola-Kirchhoff stresses over two-dimensional discretizations of the local cross sections to obtain the stress resultants and couples of the rod model. The same authors also proposed a Kelvin-Voigt type model based on a *single* viscosity parameter only and their formulation uses a vectorial strain measure defined point-wise within the cross-section, which represents parts of Biot strain tensor; see Linn *et al.* [5] for additional details.

In a recent article, Abdel-Nasser and Shabana [6] inserted a three-dimensional Kelvin-Voigt model into a geometrically nonlinear beam model using the absolute nodal coordinates formulation to obtain a viscous damping beam model. Their formulation, however, suffers from Poisson locking and is not valid for incompressible materials.

The Kelvin-Voigt model has also been used as a means of introducing numerical dissipation in dynamical equations. The dynamic balance equations of a Cosserat rod lead to an undamped, nonlinear coupled hyperbolic system of PDEs for which the formation of shock waves is possible. To overcome the numerical difficulties associated with the solution of this system, Antman [7] proposed the addition of Kelvin-Voigt type dissipative terms acting as *artificial viscosity* to achieve a *regularization* effect on the continuous model.

Linn *et al.* [5] provided a more rigorous definition of the damping parameters based on a three-dimensional continuum model, but their approach is limited to the case of a homogeneous, isotropic, and linearly elastic materials. Furthermore, their derivation is based on specific kinematic assumptions, which require cross-sections to remain plane. Lateral contractions induced by axial strains through Poisson's effects are taken into account and the model yields shear and extensional viscosities consistent with Trouton's ratio in the case of incompressible materials. Cross-sectional warping deformation, however, was ignored. Linn [8] also presented a straightforward generalization of his approach to the generalized Maxwell viscoelastic model, but the assumption inherent to the derivation do not allow applications to realistic structures involving complex geometries and material properties.

The goal of this paper is not to develop new viscoelastic constitutive laws for beams. Rather, a systematic procedure is proposed that allows existing viscoelastic models to be used within the context of beam theory.

This paper proposes a rational approach to the development of constitutive laws for viscoelastic beams. The procedure combines a three-dimensional material viscoelastic model with a three-dimensional beam theory [9, 10], which provides an exact solution of three-dimensional elasticity for static problems, but was used here as the basis for the analysis of viscoelastic, *i.e.*, dynamic problems. The applicability of the proposed methodology is limited to the dynamic analysis of lightly damped beams undergoing vibrations associated with wave lengths that are much longer than the characteristic dimensions of the cross-section. These assumptions do not put additional restrictions on the applicability of the proposed approach because beam theory for dynamics is inherently a low frequency approximation, even in the absence of viscoelastic materials.

In this effort, the beam's viscoelastic behavior was represented by the generalized Maxwell model, but other viscoelastic models could be used as well. Indeed, the present paper describes a general approach to the problem: starting from a three-dimensional viscoelastic material model, the corresponding viscoelastic beam model is constructed. It is interesting to note that the convolution integral that characterizes the generalized Maxwell model is found at the level of the one-dimensional material, three-dimensional material, and beam cross-sectional models.

The proposed approach can be generalized to viscoelastic materials featuring nonlinear material behavior; nonlinear behavior is common for elastomeric materials, for instance, even at low strain levels. In such case, the sectional analysis must be fully integrated with the solution of the beam equations, repeating the two-dimensional analysis at each time step during the simulation.

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