Dynamics of Rigid-Flexible Spatial Four-Bar Mechanism

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Flexible mechanisms are known for lightweight, less weight-to-power ratio, etc., which are desired for efficient operations of the machines having such flexible mechanisms. Various formulations for the dynamic simulation of mechanisms having both rigid and flexible links i.e., rigid-flexible systems were reported in the papers [1-3]. They presented the difficulty in the dynamic formulation due to the presence of multiple closed-loops, structural coupling, and nonlinearity, etc. This demonstrates the need for further improvements in the dynamic algorithms in terms of computational efficiency and numerical stability.

The Decoupled Natural Orthogonal Complement (DeNOC)-based formulation is proven advantageous for many systems such as serial chain systems [4] tree-type robotic systems, hyper-degrees-of-freedom systems say, ropes, chains, etc., fixed and floating base robotic systems, closed- and multi-loop rigid multibody system [5], and serial flexible multibody systems [6]. In this paper, we demonstrate the methodology with a spatial four-bar mechanism. For that, its kinematics was described first. Then, the dynamic equations of motion of the spatial mechanism were derived using DeNOC matrices for systems with rigid and flexible links. The forward dynamics and simulation of the closed-loop rigid-flexible spatial four-bar mechanism will be presented in this paper. The simulation results using the proposed methodology were compared with the results generated using the commercial software RecurDyn thus, verifying the results.

1 Methodology

In order to transmit the motion between non-parallel axes a spatial four-bar mechanism is used. It comprises of four links with one fixed link and three moving links having Revolute (R), Spherical (S), Spherical (S) and Revolute (R) joints at joint-1, joint-2, joint-4, and joint-5, respectively, as shown in Fig. 1(a). Hence, it is also called RSSR mechanism. In this mechanism, the revolute joints, joint-1 and joint-4 are in different planes, and the axes of these revolute joints are non-parallel. The RSSR mechanism has two degrees of freedom (DOF) in that one of them is redundant. This is the rotation of the coupler about its own axis. The RSSR mechanism was analyzed by cutting it at joint-5, i.e., the spherical joint, to form two subsystems, namely, two open-loop serial-chain systems, as shown in Fig. 1 (b). The resulting open-loop subsystems are the two-link serial-chain manipulator and the single-link system. The cutting of a spherical joint was intentional as it reduces the complexities of writing of equations of motion.

The cut-opened joint was then substituted with suitable constraint forces denoted with $\boldsymbol{\lambda}$, which is known as the vector of Lagrange multipliers. The Lagrange multipliers are λ_x, λ_y, and λ_z along x, y, and z-directions, respectively. These multipliers were treated as external forces to the subsystems-I and II. The dynamics of rigid-flexible spatial four-bar mechanism was formulated for the following cases:

Case 1: Rigid-Rigid-Rigid (*RRR*): In this case, the dynamic analysis was done considering all three links i.e., crank (#4), coupler (#3), and rocker (#1) as rigid (*R*). Fig. 1 shows the *RRR* RSSR mechanism.

Case 2: Rigid-Rigid-Flexible (*RRF*): In this case, the crank (#4), and the coupler (#3) were treated as rigid (*R*), whereas the rocker (#1), was considered as flexible (*F*) link.

Case 3: Flexible-Flexible-Flexible (*FFF*): Here all the three links were considered as flexible (*F*).

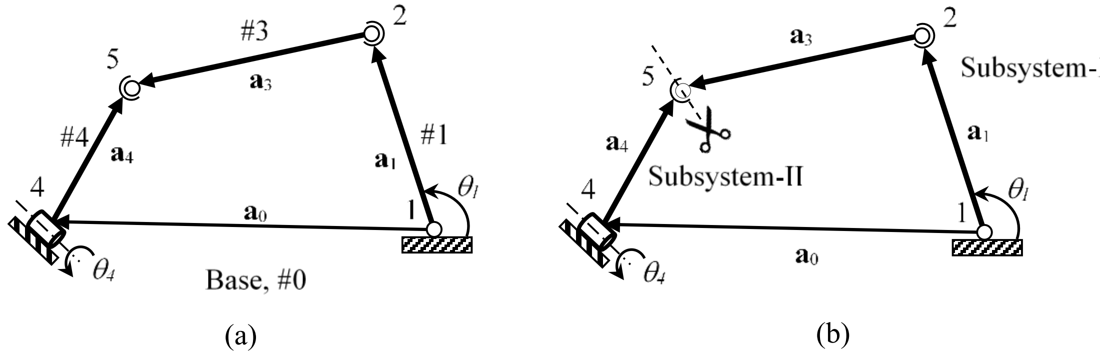

Fig. 1: RSSR mechanism

Modeling started with the formation of the DeNOC matrices for the resulting rigid-flexible subsystems by cutting the joint. For that, expressions of linear and angular velocities ($\mathbf{t}$) of all links were written in terms of joint velocities ($\dot{\theta}$). This is given by

$$\mathbf{t} = \mathbf{N}_l \mathbf{N}_d \dot{\boldsymbol{\theta}} \tag{1}$$

Matrices ($\mathbf{N}_l \mathbf{N}_d$) relating linear and angular velocities of the links to the joint rates are called the DeNOC matrices. Then, the equations of motion for each subsystem formulated as

$$\begin{aligned} \mathbf{I}^I \ddot{\mathbf{q}}^I + \mathbf{h}^I &= \boldsymbol{\tau}^I + \left(\boldsymbol{\tau}^\lambda\right)^I \\ \mathbf{I}^{II} \ddot{\mathbf{q}}^{II} + \mathbf{h}^{II} &= \boldsymbol{\tau}^{II} + \left(\boldsymbol{\tau}^\lambda\right)^{II} \end{aligned} \tag{2}$$

The procedure was illustrated through the forward dynamics of the RSSR manipulator for free fall simulation. The detailed formulation and results generated will be presented in the full paper. Finally, what has been presented here is the dynamic formulation of closed-loop rigid-flexible spatial four-bar mechanism using the DeNOC methodology and its benefits.

References

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