A model order reduction method for the simulation of gear contacts based on Arbitrary Lagrangian Eulerian formulation

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Gears are widely used to transmit torque and motion through mechanical contact interactions in fields such as automotive, aerospace and wind-energy industries. Understanding the dynamic behavior of the impacting gear pair is of paramount significance in the design process of gear systems. On the one hand, the dynamic gear forces may be many times larger than those obtained based on quasi-static conditions used in a durability problem of the gear pair [1]. The other issue is associated with the noise caused by the non-uniform rotations of the gear wheel which is the consequences of deviations from the desired geometry of the flank profiles and the flexibility of the gears[2]. Therefore, an accurate prediction of the dynamic behavior of the impact gear pair is indispensable for reliable and cost-effective gearbox design.

Gear contact problems are characterized by a large number of possible contact scenarios while only a few of them are simultaneously active at a given time. The finite element (FE) method provides a powerful tool to obtain the tooth deflections, gear mesh stiffness, and stress distribution under static conditions [3]; however, a highly refined mesh of the contact area is typically required in order to obtain convincing results. The large number of degrees of freedom (DOF) makes it impractical to use in dynamic problems, especially for simulations of long period. One way to improve efficiency is the model order reduction (MOR) technique in flexible multibody simulations. When applying the model order reduction method in gear contact problems, the addition of a static shape vector for each possibly-loaded boundary DOF will enable the contact being accurately described and the model statically complete but may be computationally expensive when the number of boundary DOF is large. On the other hand, omission of boundary DOF, though might be computationally efficient, will result in a slow convergence.

The objective of this paper is to develop a modeling technique based on Arbitrary Lagrangian Eulerian (ALE) formulation to reduce DOF while providing accurate gear meshing contact simulation. Four techniques are adopted to achieve this goal, as shown in Fig. 1. First, based on the fact that the excitation frequency of gear meshing forces is much lower than the natural frequency of the gear, thereby resulting in a quasi-static response of the gear, a low-frequency approximation is adopted in the gear model by ignoring the contribution of fixed boundary normal modes and just considering the constraint modes.

$$\mathbf{u} = \begin{bmatrix} \mathbf{u}_B \\ \mathbf{u}_I \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_C & \mathbf{\Phi}_N \end{bmatrix} \begin{bmatrix} \mathbf{q}_C \\ \mathbf{q}_N \end{bmatrix} \approx \mathbf{\Phi}_C \mathbf{q}_C \tag{1}$$

where $\mathbf{u}$, $\mathbf{u}_B$, and $\mathbf{u}_I$ are the system DOF, boundary DOF and interior DOF, respectively. $\mathbf{\Phi}_C$ and $\mathbf{\Phi}_N$ are constraint modes and fixed-boundary normal modes, and $\mathbf{q}_C$ and $\mathbf{q}_N$ the corresponding modal coordinates. Second, based on the ALE formulation, only the mesh nodes of the four engaging tooth-faces are defined as boundary nodes, resulting in a great reduction in DOF of the system. The generalized coordinates of a gear are expressed as

$$\mathbf{q} = \begin{bmatrix} \mathbf{r}^{\mathrm{T}} & \varphi^{\mathrm{T}} & \mathbf{u}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{2}$$

where $\mathbf{r}, \varphi$ are the position and orientation vector of the floating reference frame located in the center of gear. $\mathbf{u}$ are the modal coordinates, $\mathbf{u} = \begin{bmatrix} \mathbf{u}_1^{\mathrm{T}} & \mathbf{u}_2^{\mathrm{T}} & \mathbf{u}_3^{\mathrm{T}} & \mathbf{u}_4^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$, which consist of the modal coordinates of four tooth-faces. Then, dynamic equations of the flexible gear are simplified by ignoring the inertial force associated with deformation, and the calculation of Jacobian matrix is also simplified. Finally, a four-step high-efficiency contact algorithm is proposed to reduce the number of contact pairs and accelerate the detection process. To summarize, the first two techniques are aimed at reducing the DOF of the system, from $n_i + zn_b$ to zn_b by ignoring the contribution of fixed-boundary normal modes, and from zn_b to $4n_b$ by using ALE formulation, where n_i, zn_b are the number of DOF for constraint modes and fixed-boundary normal modes respectively, z is the number of teeth and n_b is the number of boundary DOF of one tooth. The third one concentrates on the simplification of dynamic equations and the fourth one is used to improve the efficiency of contact detection.

The accuracy and efficiency of the proposed method are demonstrated by simulating a typical dynamic gear contact problem and comparing the results with commercial nonlinear FE software. It takes about 180 hours to finish this task using ABAQUS, while only 8.4 hours are used to obtain comparable results with present ALE formulation. This implies that computational efficiency of the proposed method is one order of magnitude better in this typical example when compared to the conventional method.

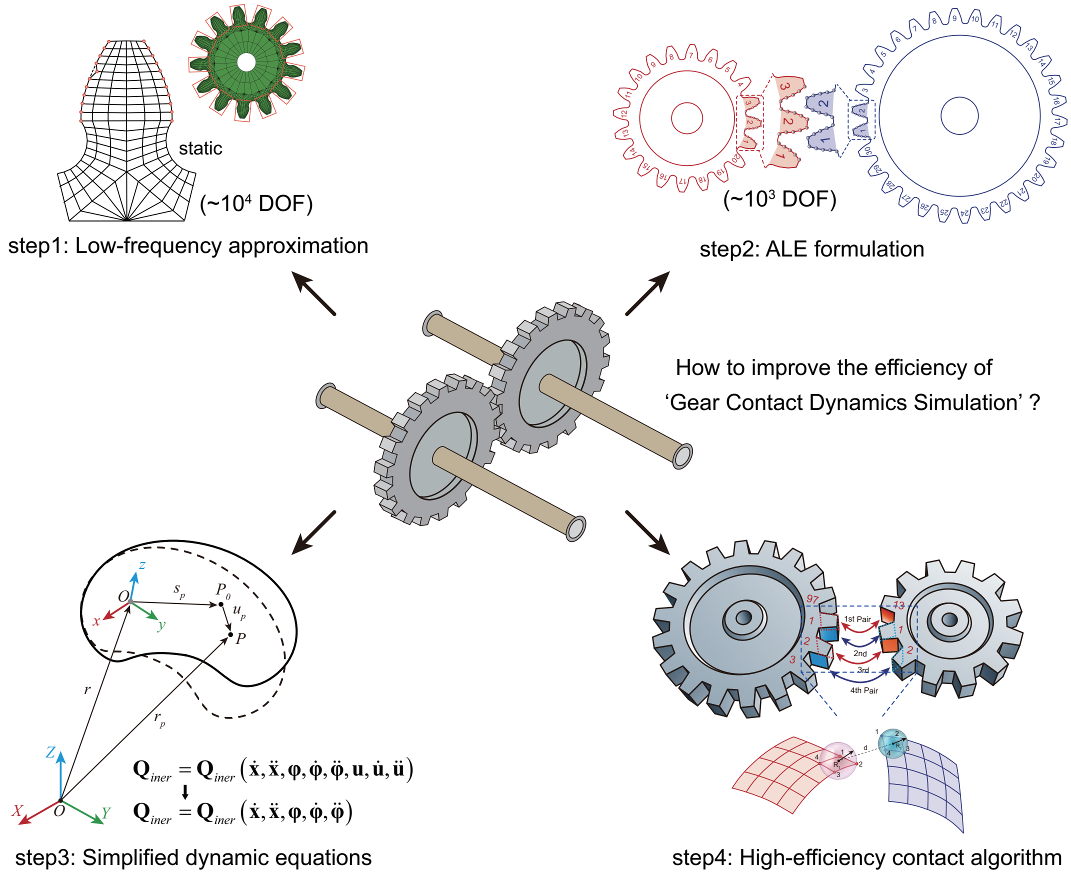

Fig. 1: Four techniques to improve the efficiency of gear contact dynamics simulation: low-frequency approximation, ALE formulation, simplified dynamic equations, and high-efficiency contact algorithm.

References

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