An Efficient Formulation for Flexible Multibody Dynamics using Dynamic Condensation of Deformation Modal Coordinates

Jong-Boo Han[1], Jin-Gyun Kim[2], and Sung-Soo Kim[3]

[1] *Graduate School of Mechanical Design and Mechatronics Engineering, Chungnam National University, jbhan83@gmail.com*

[2] *Department of Smart Machine Technology, Korea Institute of Machinery and Materials, jingyun@kimm.re.kr*

[3] *Department of Mechatronics Engineering, Chungnam National University, sookim@cnu.ac.kr*

The formulation with floating frame of reference has been widely used in the flexible multibody analysis. In this formulation, deformation of a flexible body is represented, relative to the floating reference frame. Thus this leads to the coupled equations of motion between the references coordinates which describe the gloss motion of the reference frame of the flexible body and the deformation coordinates [1]. The main advantage of this floating reference frame formulation is to apply various MOR (Model Order Reduction) techniques in order to transform the nodal coordinates into the modal coordinates for efficient analysis [2]. The most widely used MOR technique in the industry is either the technique with vibration normal modes or the technique with the combination of vibration normal modes with static correction modes such as Craig-Bampton modes. The main issue on MOR is how to select the best suited deformation modes to approximate the deformation field of the flexible body. Recently, dynamic correction of the Craig-Bampton method, called as an enhanced Craig-Bampton (ECB) method, has been developed to improve the accuracy of the approximated deformation field of a flexible body [3, 4], which is mathematically regarded as the dynamic modal condensation.

In this paper, we developed a two-step dynamic condensation method in the analysis of the flexible multibody dynamics for accurate mode selection and for efficient computation. In the first dynamics condensation, the coordinates associated with residual vibration normal modes are condensed into the coordinates associated with the selected Craig-Bampton static modes and the kept vibration normal modes for better approximation of the deformation field for the flexible body analysis. In the second dynamic condensation, the deformation modal coordinates, which are obtained from the first condensation, are again condensed into the reference coordinates which represent the gloss motion of the float reference frame of the flexible body. This leads to the reduced system equations of motion of which dimensions are 6 by the number of flexible bodies. The Eq. (1) shows the matrix and vector form of the reduced equations of motion and the associated constraint equations for the n-body flexible multibody system.

$$\begin{bmatrix} \bar{\mathbf{M}} & \bar{\mathbf{\Phi}}_{\bar{z}}^T \\ \bar{\mathbf{\Phi}}_{\bar{z}} & \bar{\mathbf{N}} \end{bmatrix} \begin{bmatrix} \ddot{\bar{\mathbf{y}}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{f}} \\ \bar{\boldsymbol{\gamma}} \end{bmatrix} \tag{1}$$

where,

$$\bar{\mathbf{M}} = diag[\mathbf{M}_1, ..., \mathbf{M}_n], \quad \mathbf{M}_i = \mathbf{M}_i^{yy} - \mathbf{M}_i^{ya}\left(\mathbf{M}_i^{aa}\right)^{-1}\mathbf{M}_i^{yaT}, (i = 1, ..., n),$$

$$\ddot{\bar{\mathbf{y}}} = [\ddot{\mathbf{y}}_1^T, ..., \ddot{\mathbf{y}}_n^T]^T, \quad \ddot{\mathbf{y}}_i = \begin{bmatrix} \ddot{\mathbf{r}}_i^T & \dot{\boldsymbol{\omega}}_i'^T \end{bmatrix}^T, (i = 1, ..., n),$$

$$\bar{\mathbf{f}} = [\mathbf{f}_1^T, ..., \mathbf{f}_n^T]^T, \quad \mathbf{f}_i = \mathbf{f}_i^y - \mathbf{M}_i^{ya}\left(\mathbf{M}_i^{aa}\right)^{-1}\mathbf{f}_i^a, (i = 1, ..., n)$$

Here, $\mathbf{M}_i^{yy}$ indicates the mass and inertia matrix associated with the reference coordinate of the body i, $\mathbf{M}_i^{aa}$ represents the modal mass matrix associated with deformation modal coordinates of the body i and $\mathbf{M}_i^{ya}$ denotes the mass and inertia coupling terms between the reference coordinates and the deformation modal coordinates of the body i. $\ddot{\mathbf{y}}_i$ is the acceleration vector associated with the reference coordinate of the body i. $\mathbf{f}_i$ is the condensed force vector associated with the reference coordinate of the body I, $\mathbf{f}_i^y$ indicates the force vector associated with the reference coordinate of the body i, and $\mathbf{f}_i^a$ denotes the force vector associated with the deformation modal coordinates of the body i. $\bar{\mathbf{\Phi}}_z$ denotes the system Jacobian matrix of the condensed system model, $\bar{\mathbf{N}}$ is the coefficient matrix of the Lagrange multiplier $\boldsymbol{\lambda}$ generated during the condensed process, and $\bar{\boldsymbol{\gamma}}$ indicates the right hand side vector of the constraint acceleration equations for the condensed system model.

After obtaining the acceleration of the floating reference frame and the Lagrange multipliers by solving Eq. (1), the acceleration of deformation modal coordinates can be determined using the following Eq. (2) for each flexible body.

$$\ddot{\mathbf{a}}_i = \left(\mathbf{M}_i^{aa}\right)^{-1}\left(\mathbf{f}_i^a - \mathbf{M}_i^{yaT}\ddot{\mathbf{y}}_i - \mathbf{\Phi}_{a_i}^{\mathrm{T}}\boldsymbol{\lambda}\right),\quad (i = 1,\cdots,n) \tag{2}$$

It is noted that the $\mathbf{M}_i^{aa}$ can be an identity matrix, if the eigenvalue analysis is applied in the preprocessing stage to the modal mass matrix and the modal stiffness matrix obtained from the first dynamic condensation. Thus, the computational cost does not increase much for the modal acceleration of each flexible body.

To verify the efficiency of the proposed formulation, we analyzed a ten-body pendulum model. Each body is considered as a flexible body with 20 beam elements. The simulation results show that the tip positions are the same from the conventional method and the proposed method as shown in Fig. 1. The computational complexity is also compared to evaluate the efficiency of the proposed method, as shown in Fig. 2. It is clearly shown that the more number of modal coordinates are used, the better efficiency is obtained using the proposed dynamic condensation method.

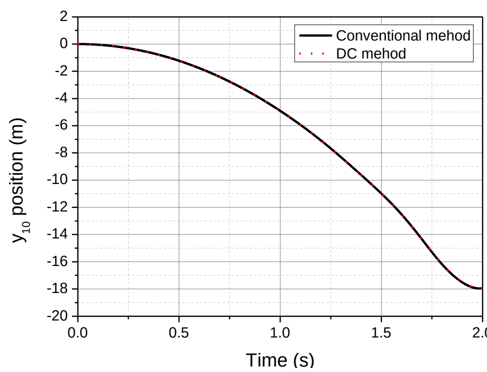

Fig. 1: Tip position (y) of 10th pendulum

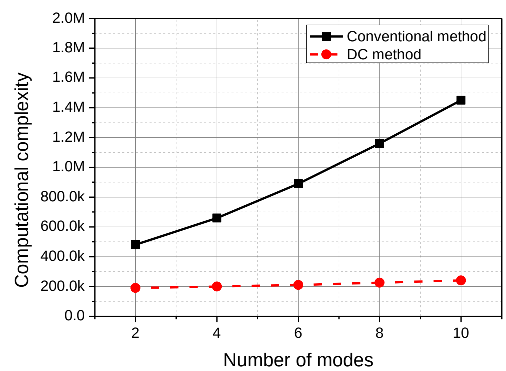

Fig. 2: Computational complecity

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