HHT Method with Velocity Constraints Violation Correction In Index 3 Equations of Motion for Multibody Systems

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Equations of motion for multibody systems in Cartesian absolute coordinates modeling method are differential-algebraic equations (DAEs)[1]

$$\begin{gathered} \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\boldsymbol{\Phi}_{\boldsymbol{q}}^{\mathrm{T}}(\boldsymbol{q})\lambda=\boldsymbol{f}(\dot{\boldsymbol{q}},\boldsymbol{q},t) \\ \boldsymbol{\Phi}(\boldsymbol{q})=\boldsymbol{0} \end{gathered} \tag{1}$$

They are index 3 DAEs[2].

The first time derivative of $\boldsymbol{\Phi}(\boldsymbol{q})=\boldsymbol{0}$ in Eq. (1) provides the velocity constraint equations as

$$\dot{\boldsymbol{\Phi}}=\boldsymbol{\Phi}_{\boldsymbol{q}}(\boldsymbol{q})\dot{\boldsymbol{q}}=\boldsymbol{0} \tag{2}$$

The Hilber-Hughes-Taylor (HHT) method is widely used in the structural dynamics community for the numerical integration of a linear set of ordinary differential equations (ODEs)[3]. HHT method is extended to the numerical integration of Eq. (1) by Dan Negrut, Rajiv Rampalli , Gisli Ottarsson and Anthony Sajdak [4]. The discretized equations are

$$\begin{aligned} \tfrac{1}{1+\alpha}\boldsymbol{M}\boldsymbol{a}_{n+1}-\left(\boldsymbol{f}-\boldsymbol{\Phi}_{\boldsymbol{q}}^{\mathrm{T}}\lambda\right)_{n+1}+\tfrac{\alpha}{1+\alpha}\left(\boldsymbol{f}-\boldsymbol{\Phi}_{\boldsymbol{q}}^{\mathrm{T}}\lambda\right)_{n}&=\boldsymbol{0} \\ \tfrac{1}{\beta h^2}\boldsymbol{\Phi}\left(\boldsymbol{q}_{n+1}\right)&=\boldsymbol{0} \end{aligned} \tag{3}$$

where

$$\begin{aligned} \boldsymbol{q}_{n+1}&=\boldsymbol{q}_n+h\dot{\boldsymbol{q}}_n+h^2\left(0.5-\beta\right)\boldsymbol{a}_n+h^2\beta\boldsymbol{a}_{n+1} \\ \dot{\boldsymbol{q}}_{n+1}&=\dot{\boldsymbol{q}}_n+h\left(1-\gamma\right)\boldsymbol{a}_n+h\gamma\boldsymbol{a}_{n+1} \end{aligned}$$

And the proposed HHT method has been released in the 2005 version of the MSC.ADAMS/Solver[4]. In this method, numerical errors due to the finite precision of the numerical integration lead to constraint violation at the velocity level. This means that $\boldsymbol{\Phi}=\boldsymbol{0}$ and $\dot{\boldsymbol{\Phi}}\neq\boldsymbol{0}$ during integration in the view of machine precision.

As for the constraints violation in numerical simulation of constrained multibody systems, Yu Qing and Cheng I-Ming[5] proposed direct violation correction method based on Moore-Penrose generalized inverse. Filipe Marques, António P. Souto and Paulo Flores[6] offered a general and comprehensive methodology to eliminate the constraints violation at the position and velocity levels for the equations of motion in the form of index 1 DAEs. For the corrected generalized velocities at each time step is

$$\dot{\boldsymbol{q}}_{n+1}^{C}=\dot{\boldsymbol{q}}_{n+1}-\left[\boldsymbol{\Phi}_{\boldsymbol{q}}^{\mathrm{T}}\left(\boldsymbol{\Phi}_{\boldsymbol{q}}\boldsymbol{\Phi}_{\boldsymbol{q}}^{\mathrm{T}}\right)^{-1}\boldsymbol{\Phi}_{\boldsymbol{q}}\dot{\boldsymbol{q}}\right]_{n+1} \tag{4}$$

In this paper, HHT method for index 3 equations of motion for multibody systems is incorporated with direct violation correction at the velocity level. Then the velocity violation can be eliminated. This means that $\boldsymbol{\Phi}=\boldsymbol{0}$ and $\dot{\boldsymbol{\Phi}}=\boldsymbol{0}$ during integration in the view of machine precision.

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