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High Order ANCF Beam Element: Integration with Computer Aided Design and Application in Leaf Spring Modeling

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A 48 DOFs high order fully parameterized ANCF beam element is proposed in this investigation. The element has 8 nodes and position and axial gradient vectors are employed as the nodal coordinates. The proposed element is consistent with the Bezier volume and the linear transformation can be performed to convert from one to the other. Cubic polynomials are used to interpolate the axial displacement field and the cross-section is described by bi-linear polynomial functions. As a result, the proposed element can be utilized to accurately describe the new designed leaf spring which has a quadratic varying thickness.

Early prior research demonstrated that the Absolute Nodal Coordinate Formulation (ANCF) finite element is consistent with NURBS geometry which is widely used in Computer Aided Design software[1]. For instance, the cable element can be converted into a cubic Bezier curve, and vice versa. The same relationship can also be built between ANCF mixed-coordinate thin plate element and bi-cubic Bezier surface. However, such a relationship cannot be established between ANCF 3D beam[2,3] element and Bezier volume because they have different degree of freedoms. A Bezier volume, which has cubic basis function at one direction, and linear basis function at the other two directions, will have 48 DOFs. A high order ANCF fully parameterized beam element is developed in this investigation so the conversion can be established. Additionally, the proposed beam element can be used in leaf spring modeling with a quadratic varying thickness. Some other details, for example, the rubber ring in the spring eye, can also be described by a few element developed in this paper.

The high order ANCF fully parameterized element has 8 nodes. Each node has the global position vector and slop vectors along the axial direction as its nodal coordinates, as Fig.1 shows. So the total DOFs of the element is 48. An arbitrary point in the element has the global position as following:

$$\mathbf{r} = \mathbf{S}\mathbf{e} \tag{1}$$

In the equation above, the nodal coordinate vector is:

$$\mathbf{e} = \left[\mathbf{r}^{1^{\mathrm{T}}}\ \mathbf{r}_x^{1^{\mathrm{T}}}\ \mathbf{r}^{2^{\mathrm{T}}}\ \mathbf{r}_x^{2^{\mathrm{T}}}\ \mathbf{r}^{3^{\mathrm{T}}}\ \mathbf{r}_x^{3^{\mathrm{T}}}\ \mathbf{r}^{4^{\mathrm{T}}}\ \mathbf{r}_x^{4^{\mathrm{T}}}\ \mathbf{r}^{5^{\mathrm{T}}}\ \mathbf{r}_x^{5^{\mathrm{T}}}\ \mathbf{r}^{6^{\mathrm{T}}}\ \mathbf{r}_x^{6^{\mathrm{T}}}\ \mathbf{r}^{7^{\mathrm{T}}}\ \mathbf{r}_x^{7^{\mathrm{T}}}\ \mathbf{r}^{8^{\mathrm{T}}}\ \mathbf{r}_x^{8^{\mathrm{T}}} \right]^{\mathrm{T}} \tag{2}$$

It can be clear see that an arbitrary point in the element has three gradient vectors, which means that the general continuum mechanics approach can be used in the proposed element.

The tensor product form of a Bezier volume which has one cubic basis function and two linear basis functions is as following[4]:

$$\mathbf{r}(u,v,w) = \sum_{i=0}^{3}\sum_{j=0}^{1}\sum_{k=0}^{1} B_{i,3}B_{j,3}B_{k,3}\mathbf{P}_{ijk}\mathbf{r}(u,v,w) = \sum_{i=0}^{3}\sum_{j=0}^{1}\sum_{k=0}^{1} B_{i,3}B_{j,3}B_{k,3}\mathbf{P}_{ijk} \tag{3}$$

In the equation above, $B_{i,n}$ is n-order Bernstein polynomial and $\mathbf{P}_{ijk}$ is the control points. So the Bezier volume has 16 control points, 48 DOFs which is the same as the high order beam element. By evaluating the position and slop vectors of the Bezier volume, it can be represented by proposed element:

$$\mathbf{e} = \mathbf{B}\mathbf{P} \tag{4}$$

In the equation, $\mathbf{e}$ is the element nodal coordinates, $\mathbf{P}$ is the control points written in vector form, and $\mathbf{B}$ is the square transformation matrix. Eq.(4) gives us a straightforward way to describe a Bezier volume by ANCF high order Beam element. If one get inverse of $\mathbf{B}$, the reverse transformation can be performed[5].

A significant difference between the proposed beam element and the traditional ANCF 3D beam element is how the thickness of the element change. Because only two nodes are employed in 3D beam element, the thickness of the element can only change linearly. But in the proposed element, the shape of the outer surfaces are defined independently. As a result, the proposed element is more flexible in modeling complex shapes.

In recent days, the leaf spring with quadratically changed thickness becomes more and more popular in vehicle suspension system because the strain created by the loads can be distributed more uniformly in the entire leaf[6,7]. The new element developed in this investigation is suitable to describe the shape of such kind of leaf spring. Besides, more details in the leaf spring such as the rubber ring in the spring eye can also be represented by the new beam element. More results will be shown in the presentation.

References

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