A planar impact model for rocking block systems

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This abstract studies the dynamics of a rectangular block rocking on a rigid plane (Fig.1). We manage to handle flat impacts involved in the rocking motion.



Fig. 1: A block rocking on a rigid plane

The flat impact owns a strong interaction variable over a contact region between two flat surfaces. A boundary layer is introduced to mimic the mechanical motion at the interface along its normal direction and adopt a local conversion coefficient to consider the energy loss (Fig.2(a) and Fig.2(b)), where $\sigma(\xi, t) = E^* \delta(\xi, t)$. In the tangential direction the interface friction is assumed to be subjected to the Coulomb friction law.



Fig.2: (a) Normally compressional displacement of the bottom surface from the base surface; (b) Strain-Stress relation at a material point _ for the boundary layer.

We then establish a set of first-order differential equations with respect to a "time-like" normal stress impulse to govern the dynamics of the flat impact.

$$\begin{cases}
m\frac{d\dot{x}}{dp^*} = f_{\tau}(t) \\
m\frac{d\dot{y}}{dp^*} = f_{\tau}(t) \\
I\frac{d\dot{\theta}}{dp^*} = m_n(t) + m_{\tau}(t)
\end{cases}$$
(1)

where x, y, θ denotes the mass center and the rotational angle, and $f_{\tau}(t), f_{\tau}(t), m_n(t), m_{\tau}(t)$ are the integrated terms of the contact stress between the block and the plane.

These differential equations can effectively describe the evolution of the interfacial stress within the flatimpact process, capture the propagation of the interface detachment, and identify the possible stick-slip transition of friction. The present study also provides a full description for the rocking dynamics, in which all the motion states of the block such as bouncing, point impact and flat impact are encapsulated into formulations.

Finally, we verify our model by the comparison with the experimental data found in existing literature (Fig.3).



Fig.3: Comparison between the simulations and the experiments for the angular responses of the four blocks in free-standing rocking motion.

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References

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