Dynamics of Multibody Systems in Fluid Flow: Geometric Formulations in Lie Group Setting

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The paper provides outline of the computationally efficient modeling and numerical simulation of dynamics of multibody system (MBS) moving in the ambient fluid. Instead of computationally expensive full discretisation of the fluid domain and solving fluid and solid dynamics on different meshes - that is most common setting for fluid-structure-interaction simulation tasks - our approach is based on specifically tailored methodology that comprises analytical and numerical modeling techniques as well as model reduction procedures rooted in geometric mechanics formulations. Dynamics simulation is based on the fully coupled mathematical model that is derived via consecutive geometric procedures - before any discretisation process is undertaken - reducing thus its dimensionality without compromising any accuracy. Moreover, since geometric reductions treat the system in a monolithic way by taking advantages of the inherent symmetries [1], the procedure provides a convenient platform for assuring that underlying momentum maps (i.e. conservation laws [2]) are also preserved in the dicretised form. Furthermore, dynamics is to be solved within the same time-integration procedure without need to separate different media, meaning that much less computational power is involved in determination of the dynamic response. It should be noted that the abovementioned approach is specially relevant if dynamics of the submerged solids moving through the ambient fluid is of the primary interest. Thus, dynamics of the flow is solved only to determine its effect on the moving solids, while no full discretisation of the fluid domain is taking place.

First study in the paper is focused on dynamics of MBS moving in the perfect fluid. It is assumed that a submerged system comprises blunt bodies and sets the ambient fluid into motion without generating circulation. Hence, flow model is potential flow of an ideal fluid - inviscid, incompressible fluid with irrotational flow around no-sharp-edges kinematical chain (flow with zero circulation). By using principles of geometric mechanics - under assumption of its Hamiltonian structure - it is possible to derive reduced motion equations of the MBS-fluid system in a monolithic way i.e. without separating its fluid and solid part.

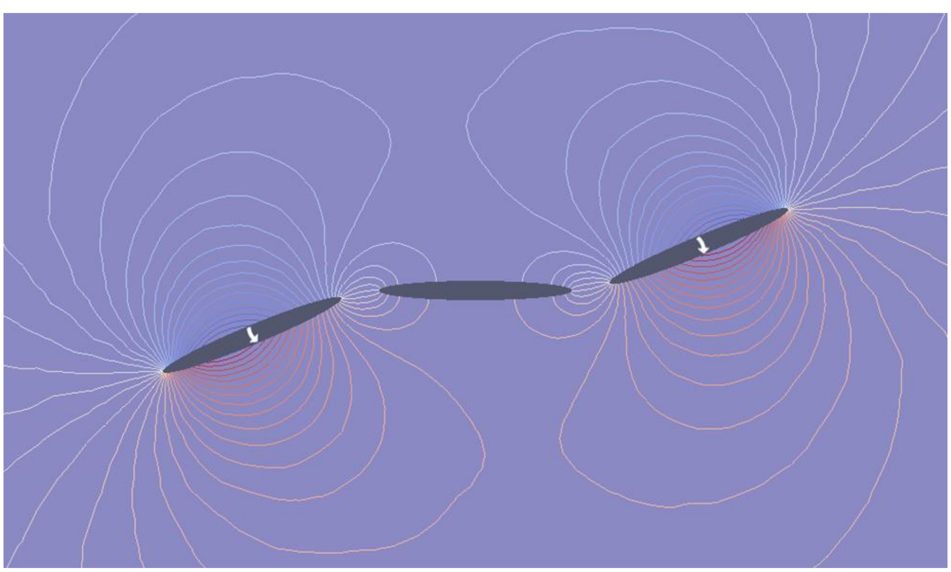

Fig. 1: Velocity potential of the flow around submerged MBS

To this end, we use Lie group setting and identify two symmetries for further proceeding. The first reduction exploits particle relabeling symmetry, associated with the conservation of circulation (here set to zero) for incompressible ideal fluids, modeled as volume-preserving diffeomorphism Lie group. Fluid kinetic energy, fluid Lagrangian and associated momentum map are invariant with respect to this symmetry [3]). Consequently, the equations of motion for the submerged MBS can be formulated without explicitly incorporating the fluid variables, while effect of the fluid flow to MBS overall dynamics is accounted for by the added masses to the submerged bodies [4]. In such approach, the added masses are expressed as boundary integral functions of the fluid density and the flow velocity potential. After particle relabeling symmetry, further reduction of the system is associated with the symmetry based on invariance of the dynamics under superimposed rigid motions [5].

In the next study, we relax assumption of non-circulatory flow around the system of solids and allow for inclusion of sharp-edged body that generates non-zero amount of circulation. We can keep basic assumption of the ambient fluid potential flow and pursue two stages reduction of the fluid-solid system - i.e symplectic reduction based on particle relabeling symmetry and Lie-Poisson reduction of superimposed rigid motions - similarly as in the example above. However, here symplectic reduction needs to involve specification of the fluid vorticity field [6], and, consequently, vortex-shedding mechanism that takes into account fluid viscous effects must be included. In this study, this will be done numerically by imposing Kutta condition on the body sharp trailing edge [7].

In order to calculate dynamics of the fully coupled fluid-solid system, numerical integration scheme that operates in Lie group setting is specifically designed. This scheme possess the same structure as MBS state space Lie group time-integrator described in [8]. However, in order to account for the added masses effect of the reduced fluid-solid monolithic system, integrator is supplemented with the added masses determination procedure based on the properties of fluid flow, and calculated via boundary element method.

Acknowledgments

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